Sphere

SAS.

5th sem unit-1

① Find the radius & Centre of sphere $x^2+y^2+z^2-2x+4y-6z=2$

② Find eqn of sphere passing the line joining points $A(2,-3,4)$ $B(-5,6,-7)$ as Diameter

LA ③ Find limiting poits of Co-axial System $x^2+y^2+z^2-20x+30y-40z+29+\lambda(2x-3y+4z)=0$.

④ Two Spheres of raddi r_1, r_2 cut orthogonally. prove that radius of common circle is $\frac{r_1 r_2}{\sqrt{r_1^2+r_2^2}}$

LA ⑤ Find the limiting points ~~dfin~~ of Co-axial system defined by spheres $x^2+y^2+z^2+3x-3y+6=0$ and $x^2+y^2+z^2-6y-6z+6=0$.

⑥ find center and radius of sphere. $2x^2+2y^2+2z^2-2x+4y+2z+3=0$.

⑦ find condition that the spheres $a(x^2+y^2+z^2)+2lx+2my+2nz+p=0$ and $b(x^2+y^2+z^2)=k^2$ may cut orthogonally.

Long మళ్ళీ Chance

రావడం So ఇవి prepare చేస్తే better. SAS లో Actual LAS. Quetions easy రావడం

Sphere కనుక్కోవడం ← Quetions → points కనుక్కోవడం, proof

① find the equation of sphere through four points $(0,0,0)$, $(-a,b,c)$, $(a,-b,c)$ $(a,b,-c)$

② find equation of sphere which passes points $(0,0,0)$ $A(-1,2,3)$ $B(1,-2,3)$ $C(1,2,-3)$

③ find equation of sphere having its centre on the line $2x-3y=0=5y+2z$ and passing through points $(0,-2,-4)$ & $(2,-1,-1$

④ find equation to a sphere passing through the points $(1,-3,4)$, $(1,-5,2)$ $(1,-3,0)$ and having centre on plane $x+y+z=0$.

⑤ find points of intersection of line $2x-1=y+3=-z+4$ with S $x^2+y^2+z^2-6x+8y-4z+4$

⑥ if any tangent plane to sphere $x^2+y^2+z^2=r^2$ makes intercepts a, b and c on the co-ordinate axes PT $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{r^2}$

⑦ find eqn of sphere that passes $(0,3,0)$ $(-2,-1,-4)$ & cuts orthogonally the 2 spheres $x^2+y^2+z^2+x-3z-2=0$

$2(x^2+y^2+z^2)+x+3y+4=0$.

⑧ find limiting points of co-axil system by spheres

$x^2+y^2+z^2+4x-2y+2z+6=0$

$-2z+6=0$

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unit-1 Sphere

① what is sphere and write its equation.

The locus of a point which is is at a constant distance from a fixed point is called as Sphere.

The eqn of Sphere is,

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0$$

where x, y, z variables

u, v, w, d are parameters

$$u^2+v^2+w^2-d \geq 0$$

$$\boxed{\sqrt{u^2+v^2+w^2-d} = \text{radius}}$$ of sphere.

② find radius and centre of Sphere

$$x^2+y^2+z^2-2x+4y-6z=2$$

given $x^2+y^2+z^2-2x+4y-6z-2=0$ — ①

Comparing ① with Square eqn

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0$$

$2u=-2$; $2v=4$; $2w=-6$; $d=-2$

$u=-1$; $v=2$; $w=-3$ $d=-2$

Centre $c(-u,-v,-w) = (1,-2,3)$

Radius $r = \sqrt{u^2+w^2+w^2-d}$

$= \sqrt{(-1)^2+2^2+(-3)^2-(-2)}$

$= \sqrt{1+4+9+2} = \sqrt{16} = 4$

Radius $\boxed{r=4}$

$c(1,-2,3)$ is centre of given sphere.

③ find centre and radius of sphere, ✓

$$2x^2+2y^2+2z^2-2x+4y+2z+3=0$$

Given sphere is ↓

$$2\left(x^2+y^2+z^2-x+2y+z+\frac{3}{2}\right)=0$$ ✓

$$x^2+y^2+z^2-x+2y+z+\frac{3}{2}=0 \quad -①$$

⇒ comparing eqn ① with general sphere eqn $\boxed{x^2+y^2+z^2+2ux+2vy+2wz+d=0}$

$2u=-1$; $2v=2$; $2w=1$; $d=\frac{3}{2}$

$u=-\frac{1}{2}$; $v=1$; $w=\frac{1}{2}$;

Centre $(-u,-v,-w)=\left(\frac{1}{2},-1,\frac{-1}{2}\right)$

$r=\sqrt{u^2+v^2+w^2-d}$

Radius $r=\sqrt{\left(-\frac{1}{2}\right)^2+(1)^2+\left(\frac{+1}{2}\right)^2-\frac{3}{2}}=\sqrt{\frac{1}{4}+1+\frac{1}{4}-\frac{3}{2}}$

$r=\sqrt{\frac{2}{4}+1-\frac{3}{2}}=\sqrt{\frac{3}{2}-\frac{3}{2}}=\sqrt{0}=0$

$\boxed{r=0}$

Extra Q :-

find equation of sphere having circle $x^2+y^2+z^2+10y-4z-8=0$, $x+y+z=3$ as great circle.

Given eqn of greater circle $x+y+z=3$

given circle, $x^2+y^2+z^2+10y-4z-8=0$

eqn of sphere through circle is

Eqn of circle + K (eqn greatcircle) = 0

Tip :- every sphere lies on its greater circle so, Circle + k (great circle) = 0 is sphere circle $S + K$ (gratu circle)

$$x^2+y^2+z^2+10y-4z-8+k(x+y+z-3)=0$$

$$x^2+y^2+z^2+10y-4z+kx+ky+kz-3k-8=0$$

$$x^2+y^2+z^2+kx+(10+k)y+(-4+k)z-3k-8=0 \quad ①$$

comparing with

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0$$

$2u=k \qquad 2v=10+k \qquad 2w=-4+k$

$u=\frac{k}{2} \qquad v=\frac{10+k}{2} \qquad w=\frac{k-4}{2}$

Centre $\left(\frac{-k}{2}, \frac{-(10+k)}{2}, \frac{-(k-4)}{2}\right)$ lies on great circle

$$x+y+z-3=0$$

$$\Rightarrow \frac{-k}{2}-\frac{(10+k)}{2}-\frac{k-4}{2}-3=0$$

$$\frac{-k-10-k-k+4-6}{2}=0 \Rightarrow -3k-12=0$$

$$-3k=12$$

$$\boxed{k=-4}$$

Sub $k=-4$ in ①

$$x^2+y^2+z^2-4x+(10-4)y+(-4-4)z-3(-4)-8=0$$

$+12-8$

$x^2+y^2+z^2-4x+6y-8z+4=0$ is sphere equation.

⑤ find condition that the spheres $a(x^2+y^2+z^2)+2lx+2my+2nz+p=0$ and $b(x^2+y^2+z^2)=k^2$ may cut orthogonally.

Given Spheres

$$a(x^2+y^2+z^2)+2lx+2my+2nz+p=0 \quad — ①$$

$$b(x^2+y^2+z^2)=k^2 \Rightarrow x^2+y^2+z^2-\frac{k^2}{b}=0$$

comparing ① with can written as

$$a\left[x^2+y^2+z^2+2\frac{l}{a}x+\frac{2m}{a}y+\frac{2n}{a}z+\frac{p}{a}\right]=0$$

Comparing with $x^2+y^2+z^2+2u_1x+2v_1y+2w_1z+d=0$

$$u_1=\frac{l}{a}, \quad v_1=\frac{m}{a} \quad w_1=\frac{n}{a} \quad d=\frac{p}{a}$$

Comparing 2nd eqn with general eqn

$$u_2=0 \quad v_2=0 \quad w_2=0 \quad d_2=\frac{-k^2}{b}$$

two spheres cut orthogonally if.

$$2u_1v_2+2v_1v_2+2w_1w_2=d_1+d_2$$

$$2\left(\frac{l}{a}\right)(0)+2\left(\frac{m}{a}\right)(0)+\frac{n}{a}(0)=\frac{p}{a}-\frac{k^2}{b}$$

$$0=\frac{p}{a}-\frac{k^2}{b} \Rightarrow \frac{p}{a}=\frac{k^2}{b} \Rightarrow pb=k^2a.$$

$\Rightarrow \boxed{ak^2=bp}$ is condition for orthogonality of given 2 spheres.

extra Q :-

① A variable plane through a fixed point (a,b,c) cut the co-ordinate axes in the point A, B, C show that the locus of the centres of the sphere OABC is $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$.

Note [Same Question & a,b,c placeలో p,q,r [illegible]]

Let the equation of sphere *OABC* be,

$$\boxed{x^2 + y^2 + z^2 + 2ux + 2vy + 2wz = 0} - ①$$

And the points *A*, *B*, *C* cuts the co-ordinate axes ar

$$(-2u, 0, 0), (0, -2v, 0), (0, 0, -2w)$$

The equation of plane which makes intercepts on th

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

$$\Rightarrow \quad \frac{x}{-2u} + \frac{y}{-2v} + \frac{z}{-2w} = 1$$

Since, plane passes through fixed point (a, b, c)

$$\frac{a}{-2u} + \frac{b}{-2v} + \frac{c}{-2w} = 1 \quad - ②$$

Let, (x, y, z) be centre of equation (1),

Centre of sphere is;

$$C = (-u, -v, -w)$$

$$\Rightarrow (x, y, z) = (-u, -v, -w)$$

from – ②

$$\frac{a}{2(-u)} + \frac{b}{2(-v)} + \frac{c}{2(-w)} = +1$$

$$\frac{a}{2x} + \frac{b}{2y} + \frac{c}{2z} = 1$$

$$\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$$

② Find equation of sphere passing through line joining points $A(2,-3,4)$ $B(-5,6,-7)$ as diameter

Formula :- eqn of sphere passing through line joining points $A(x_1,y_1,z_1)$ & $B(x_2,y_2,z_2)$ as diameter is

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)+(z-z_1)(z-z_1)=0 \quad ①$$

we have $A(x_1,y_1,z_1) = A(2,-3,4)$

$B(x_2\ y_2,z_2) = B(-5,6,-7)$

Sub points in eqn ①

$$(x-2)(x-(-5))+(y-(3))(y-(6))+(z-4)(z-(-7))$$

$$(x-2)(x+5)+(y+3)(y-6)+(z-4)(z+7)$$

$$x^2+5x-2x-10+y^2-6y+3y-18+z^2+7z-4z-28$$

$$x^2+y^2+z^2+3x-3y+3z-56=0$$

extra Q or find eqn of sphere passing origin & cuts intercepts on axes at A, B, C.

① A sphere of constant radius K passes through the origin and cuts the axes in A, B, and C Find locus of Centroid of Triangle.

given Sphere pases through origin and cuts axes in A, B, C.

let A(a,0,0) B(0,b,0) C(0,0,C)

& given radius = K. of Sphere. OABC where, O is origin (0,0,0)

let

⇒ eqn of Sphere is

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0 \quad -①$$

① passes through origin (0,0,0) ⇒ Sub in ①

$$0^2+0^2+0^2+2u(0)+2v(0)+d=0$$

$$\boxed{d=0}$$

eqn ① passes through A(a,0,0)

$$\Rightarrow a^2+0+0+0+2u(a)+2v(0)+2w(0)+0=0$$

$$a^2+2ua=0 \Rightarrow 2ua=-a^2 \Rightarrow \boxed{u=\frac{-a}{2}}$$

eqn ① passes through B(0,b,0)

Sub in ①

$$\Rightarrow 0^2+b^2+0^2+2v(0)+2v(b)+2w(0)+0=0$$

$$b^2+2vb=0 \Rightarrow \boxed{v=\frac{-b}{2}}$$

eqn ① passes through C(0,0,c) & d=0

Sub in ① ⇒ $0^2+0^2+c^2+2u(0)+2w(c)=0$

$$c^2+2wc=0 \Rightarrow \boxed{w=\frac{-c}{2}}$$

Sub d=0, $u=\frac{-a}{2}$, $v=\frac{-b}{2}$, $w=\frac{-c}{2}$ in ①

$$x^2+y^2+z^2+2(\frac{-a}{2})x+2(\frac{-b}{2})y+2(\frac{-c}{2})z+0=0$$

$$x^2+y^2+z^2-ax-by-cz=0 \quad -②$$

So ∴ eqn of Sphere OABC is

$$x^2+y^2+z^2-ax-by-cz=0.$$

Note only sphere eqn adigite [illegible] ① [illegible] వరకు రాయాలి.

locus of centroid of Triangle ABC :-

Radius $r=\sqrt{u^2+v^2+w^2-d}$

from ① $u=-\frac{1}{2}a$, $v=-\frac{1}{2}b$, $w=-\frac{1}{2}c$, $d=0$

$$r=\sqrt{(-\tfrac{1}{2}a)^2+(-\tfrac{1}{2}b)^2+(-\tfrac{1}{2})^2c+0}$$

$$r=\sqrt{(\tfrac{+1}{4})[a^2+b^2+c^2}$$

$$\text{radius}=k=\frac{1}{2}\sqrt{a^2+b^2+c^2}.$$

$$2k=\sqrt{a^2+b^2+c^2}\Rightarrow 4k^2=a^2+b^2+c^2 \text{ — ③}$$

Let centroid ~~through~~ of (A, B, C) Trangle is (x_1, y_1, z_1)

$A(a,0,0)$ $B(0,b,0)$ $C(0,0,c)$

$$x_1=\frac{a+0+0}{3},\quad y_1=\frac{b+0+0}{3},\quad z_1=\frac{0+0+c}{3}$$

$$x_1=\frac{a}{3},\quad y_1=\frac{b}{3},\quad z_1=\frac{c}{3}.$$

$$\Rightarrow a=3x_1,\quad b=3y_1,\quad c=3z_1$$

Sub a, b, c in eqn ③

$$4k^2=(3x_1)^2+(3y_1)^2+(3z_1)^2$$

$$4k^2=9(x_1^2+y_1^2+z_1^2)$$

So $9(x_0^2+y_0^2+z_0^2)=4k^2$ is locus of centroid of ΔABC.

⑤ find eqn of Sphere through the points (0,0,0), (-a, b, c) (a, -b, c) and (a, b, -c) find radius.

given points O(0,0,0) A(-a,b,c) B(a,-b,c) C(a,b,-c)

let eqn of Sphere be

$x^2+y^2+z^2+2ux+2vy+2wz+d=0$ — ①

eqn ① passes Through O(0,0,0)

$\Rightarrow 0^2+0+0+d=0 \Rightarrow d=0$

eqn ① passes Through A(-a,b,c)

$(-a)^2+b^2+c^2+2u(-a)+2v(b)+2w(c)+d=0$

$d=0 \Rightarrow$ $a^2+b^2+c^2-2au+2vb+2wc=0$ — ②

eqn ① passes Through B(a,-b,c)

$(a^2)+(-b)^2+c^2+2u(a)+2v(-b)+2w(c)+d=$

$d=0 \Rightarrow$ $a^2+b^2+c^2+2au-2bv+2wc=0$ — ③

eqn ① pass Through C(a,b,-c)

$a^2+b^2+(c)^2+2u(a)+2v(b)+2w(-c)+d=$

$a^2+b^2+c^2+2au+2bv-2wc=0$ — ④

add→Solving ② & ③

$a^2+b^2+c^2-2au+2bv+2cw=0$

(+) $a^2+b^2+c^2+2au-2bv+2cw=0$

$2a^2+2b^2+2c^2+4cw=0$

$\Rightarrow 4cw = -2(a^2+b^2+c^2)$

$$\boxed{w = \frac{-(a^2+b^2+c^2)}{2c}}$$ — ⑤

add, ③ & ④

$a^2+b^2+c^2+2au-2bv+2wc$

$a^2+b^2+c^2+2au+2bv-2wc$

$2a^2+2b^2+2c^2+4au=0$

$2[(a^2+b^2+c^2)+2au]=0$

$2\neq0$

$2au = -(a^2+b^2+$

$$\boxed{u = \frac{-(a^2+b^2+c^2)}{2a}}$$

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eqn ③ is $a^2+b^2+c^2+2au-2bv+2cw=0$

Sub w ~~we got values~~ in it

$$\Rightarrow a^2+b^2+c^2+2a\left(\frac{-(a^2+b^2+c^2)}{2a}\right)-2bv+2c\left(-\frac{(a^2+b^2+c^2)}{2c}\right)=0$$

$$a^2+b^2+c^2-(a^2+b^2+c^2)-2bv-(a^2+b^2+c^2)=0$$

$$-2bv=+(a^2+b^2+c^2)$$

$$\Rightarrow v=-\frac{(a^2+b^2+c)}{2b} \quad -⑦$$

Substituting eqn ⑤ ⑥, ⑦ in ①

2 u x

$$x^2+y^2+z^2+2\left(\frac{-(a^2+b^2+c^2)}{2a}\right)x+2\left(\frac{-(a^2+b^2+c^2)}{2b}\right)y+2\left(-\frac{(a^2+b^2+c^2)}{2c}\right)$$

$$x^2+y^2+z^2-(a^2+b^2+c^2)\left[\frac{x}{a}+\frac{y}{b}+\frac{z}{c}\right]=0$$

$$\frac{x^2+y^2+z^2}{a^2+b^2+c^2}-\frac{x}{a}-\frac{y}{b}-\frac{z}{c}=0$$

Radius :-

$$r=\sqrt{u^2+v^2+w^2-d}.$$

$$r=\sqrt{-\left[\frac{a^2+b^2+c^2}{2a}\right]^2+\left[\frac{-(a^2+b^2+c^2)}{2b}\right]^2+\left[\frac{-(a^2+b^2+c^2)}{2c}\right]}$$

$$r=\sqrt{\left(\frac{1}{4a^2}+\frac{1}{4b^2}+\frac{1}{4c^2}\right)\left[a^2+b^2+c^2\right]^2}$$

$$r=\frac{a^2+b^2+c^2}{2}\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}.$$

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⑥ find equation of Sphere which passes through the points O(0,0,0) A(−1,2,3) B(1,−2,3) C(1,2,−3)

Given points are

O(0,0,0) A(−1,2,3) B(1,−2,3) C(1,2,−3)

Let the equation of Sphere be,

$x^2+y^2+z^2+2ux+2vy+2wz+d=0$ — ①

eqn ① passes Through O(0,0,0)

$0+0+0+0+0+0+d=0 \Rightarrow \boxed{d=0}$

eqn ① passes Through A(−1,2,3)

$(-1)^2+2^2+3^2+2u(-1)+2v(2)+2w(3)+d=0$

$1+4+9-2u+4v+6w+0=0$

$-2u+4v+6w=-14$

$u-2v-3w-7=0$ — ②

Eqn ① passes Through B(1,−2,3).

$\Rightarrow 1^2+(-2)^2+3^2+2u(1)+2v(-2)+2w(3)+d=0$

$1+4+9+2u-4v+6w=0$

$2u-4v+6w+14=0 \Rightarrow u-2v+3w+7=0$ — ③

eqn ① passes Through C(1,2,−3)

$1^2+2^2+(-3)^2+2u(1)+2v(2)+2w(-3)+d=0$

$1+4+9+2u+4v-6w+0=0$

$2u+4v-6w+14=0 \Rightarrow 2(u+2v-3w+7)=0$

$2\neq0 \Rightarrow u+2v-3w+7=0$ — ④

Solving ② & ③

$u-2v-3w-7=0$

$u-2v+3w+7=0$

(−) (+)

~~2u~~ $0+0-6w-14=0$

$6w=-14 \Rightarrow \boxed{w=-7}$

Solving ③ & ④.

$u-2v+3w+7=0$

$u+2v-3w+7=0$

$2u+0+0+14=0$

$2u=-14 \Rightarrow \boxed{u=-7}$

Substitute u, w in any of eq^ns
② ③ , ④

$\Rightarrow u-2v+3w+7=0$.

⓪ $-7-2v+3\left(-\frac{7}{3}\right)+7=0$

$-7-2v-7+7=0$

$-2v=7 \Rightarrow \boxed{v=\frac{-7}{2}}$

$d=0$

Sub $u=-7$, $v=-\frac{7}{2}$, $w=\frac{-7}{3}$ in eq^n ①.

$x^2+y^2+z^2+2(-7)x+2\left(-\frac{7}{2}\right)y+2\left(-\frac{7}{3}\right)z=0$

$x^2+y^2+z^2=14x-7y-\frac{14}{3}z=0$

$3(x^2+y^2+z^2)-42x-21y-14z=0$. is eq^n of Sphere.

⑥ find eq^n of Sphere having centre on line $2x-3y=0=5y+2z$ and passing through points $(0,-2,-4)$ & $(2,-1,-1)$

Given eq^n of Sphere passes Through points $A(0,-2,-4)$ $B(2,-1,-1)$ and centre lies on line $2x-3y=0=5y+2z$.

let eq^n of Sphere is

$x^2+y^2+z^2+2ux+2vy+2wz+d=0$ — ①

$\Rightarrow$ ① passes Through $A(0,-2,-4)$

$0^2+(-2)^2+(4)^2+2u(0)+2v(-2)+2w(-4)+d=0$

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$0 + 4 + 16 + 0 - 4v - 8w + d = 0.$

$20 - 4v - 8w + d = 0$

$4v + 8w - d - 20 = 0$ -②.

eqn ① pass through point $B(2,-1,-1)$

$2^2 + (-1)^2 + (-1)^2 + 2u(2) + 2v(-1) + 2w(-1) + d = 0.$

$4 + 1 + 1 + 4u - 2v - 2w + d = 0.$

$4u - 2v - 2w + d + 6 = 0$ -③

given that centre of sphere ① lies on line

$5y + 2z = 0 = 2x - 3y$ i.e $c(-u,-v,-w)$

Sub c in line.

$\Rightarrow 5(-v) + 2(-w) = 0 = 2(-u) - 3(-v).$

$-5v - 2w = 0$ -④

$-2u + 3v = 0$ -⑤.

Solving.

$-4v-8w+d+20=0$ — ①

$4u-2v-2w+d+6=0$ — ②

$-2u+3v=0$ — ③

$-5v-2w=0$ — ④

① − ②

$-4v-8w+d+20=0$

$-4u-2v-2w+d+6=0$

$+ \quad + \quad - \quad -$

$-4u-2v-6w+14=0$ — ⑤

Solving eqn ③ × 2 & ⑤ — substraction

$-4u+6v=0$

$-4u-2v-6w+14=0$

$+ \quad + \quad + \quad -$

$8v+6w-14=0$ — ⑥

Solving ⑥ and ④

eqn ~~⑥~~ ④ × 3 and ⑥

$-15v-6w=0$

$8v+6w-14=0$

$-7v-14=0$

$-7v=14 \Rightarrow -v=2 \Rightarrow \boxed{v=-2}$

Subst $v=-2$ in eqn ③ & ④

$-2u+3v=0$ & $-5v-2w=0$

$-2u+3(-2)=0$ & $-5(-2)-2w=0$

$-2u-6=0$ $\quad$ $10-2w=0$

$-2u=+6$ $\quad$ $10=2w$

$\boxed{u=-3}$ $\quad$ $\boxed{w=5}$

$\boxed{v=-2}$

Sub u, v, w in eqn ①

$-4v-8w+d+20=0$

$-4(-2)-8(5)+d+20=0$

$8-40+d+20=0$

$d-12=0$

$\boxed{d=12}$

eqn of sphere $x^2+y^2+z^2+2(-3)x+2(-2)y+2(5)z+12=0$

$\boxed{x^2+y^2+z^2-6x-4y+10z+12=0}$

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⑨ find eqn to Sphere passing Through points $(1,-3,4)$ $(1,-5,2)$ $(1,-3,0)$ and having centre on plane $x+y+z=0$

given point $A(1,-3,4)$ $B(1,-5,2)$ and $C(1,-3,0)$.

let eqn of Sphere is.

$x^2+y^2+z^2+2ux+2vy+2wz+d=0$.

eqn ① passes Through $A(1,-3,4)$.

$1+9+16+2u-6v+8w+d=0$

$2u-6v+8w+d=+26=0$ ②

eqn ① passes Through $B(1,-5,2)$

$2u-10v+4w+d+30=0$ ③.

eqn ① passes Through $C(1,-3,0)$

$2u-6v+d=+10=0$ ④.

Solving ② & ④

$2u-6v+8w+d+26=0$

$\underline{\overset{-}{2u}\overset{+}{-6v}\quad\overset{-}{+d}\overset{-}{+10}=0}$

$8w+16=0$.

$8w=-16$

$\boxed{w=-2}$

Solving ③ & ④

$2u-10v+4w+d=+30=0$

$\underline{\overset{-}{2u}\overset{+}{-6v}+\overset{-}{0}\overset{+}{+d}\overset{-}{+10}=0}$

$-4v+4w+20=0$

$-4v$ ~~$+4(-2)+20$~~ $=0$

$-4v-4+20=0 \Rightarrow -4v=-16$

$\boxed{v=4}$

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$-4v+4w+20=0 \Rightarrow$ we have $w=-2$

$\Rightarrow -4v+4(-2)+20=0 \Rightarrow -4v-8+20=0$

$-4v=-12$

$\boxed{v=3}$

sub $w=-2$, $v=3$ in ~~any~~ eqn below

Actually given that centre lies on plane $x+y+z=0$

Centre $(-u,-v,-w)$ lies on above eqn

$\Rightarrow -u-v-w=0$

$u+v+w=0$ sub $v=3, w=-2$

$\Rightarrow u+3-2=0 \Rightarrow \boxed{u=-1}$

sub u, v, w in any of ②, ③, ④ eqn.

$\Rightarrow$

$2u-6v+8w+d+26=0$

$2(-1)-6(3)+8(-2)+d+26=0$

$-2-18-16+d+26=0$

$-36+26+d=0$

$\boxed{d=+10}$

Sub $u=-1$, $v=3$, $w=-2$, $d=10$ in eqn ①

$x^2+y^2+z^2+2(-1)x+2(3)y+2(-2)z+10=0$

$x^2+y^2+z^2-2x+6y-4z+10=0$

is eqn of sphere with given conditions.

If asked to prove ⇒ U just compare co-efficients $k = 1, -1 \Rightarrow$ consistent so ⇒ circles lie on same sphere

⑩ ~~Pr~~ the circles $x^2+y^2+z^2-2x+3y+4z-5=0$, $5y+6z+1=0$; $x^2+y^2+z^2-3x-4y+5z-6=0$, $x+2y-7z=0$ lie on same sphere and find it equations.

Given,

eqn of 1st circle

$$x^2+y^2+z^2-2x+3y+4z-5=0, \quad 5y+6z+1=0$$

eqn of sphere through 1st circle is

$$C_1 + KP_1 = 0$$

$$x^2+y^2+z^2-2x+3y+4z-5+K(5y+6z+1)=0$$

$$x^2+y^2+z^2-2x+(3+5K)y+(4+6K)z+K-5=0 \quad \text{①}$$

eqn of 2nd circle through is

$$x^2+y^2+z^2-3x-4y+5z-6=0, \quad x+2y-7z=0$$

$C_2 + K'P_2$ is sphere through 2nd circle.

$$x^2+y^2+z^2-3x-4y+5z-6+K'(x+2y-7z)=0$$

$$x^2+y^2+z^2-(3-K')x-(4-2K')y+(5-7K')z-6=0 \quad \text{②}$$

when the given 2 circles lie on same sphere ⇒ Then the eqn ① & ② represents eqn of same sphere so equating co-efficients of ① & ②

x coefficient	y coefficient	~~z-~~coff.
$-2 = -(3-k')$	$3+5K = -(4-2K')$	~~4~~
$2 = 3-K'$	$3+4+5K = +2K'$	$\boxed{K=1}$
$k' = 3-2 = 1$	~~8~~ $7+5K = 2(1)$	$\boxed{K=-1}$
$\boxed{K'=1}$	$5K = 2-7$	
	$K = \frac{-5}{5} = -1$	

K, K' values are consistent which shows that 2 circles lie on same sphere.

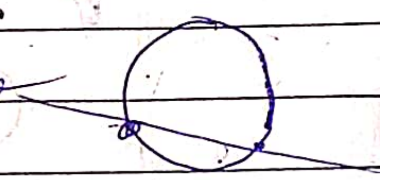
∴ Sub $K=-1$ or $K'=1$ in any of ① & ②
we get sphere.

Sub in ① $x^2+y^2+z^2-2x+(3+5K)y+(4+\cancel{4K})z+K-5=0$
$K=-1$

$x^2+y^2+z^2-2x+(3+5(-1))y+(4+6(-1))z+-1-5=0$
$x^2+y^2+z^2-2x-2y-2z-6=0$ is
eqn of sphere.

⑱ find points of intersection of
line $2x-1=y+3=-z+4$ with sphere
$x^2+y^2+z^2-6x+8y-4z+4=0$

given eqn of line



$2x-1=y+3=-z+4$
Sphere $x^2+y^2+z^2-6x+8y-4z+4=0$ — ②

let $2x-1=y+3=-z+4=r$
$2x-1=r$; $y+3=r$; $-z+4=r$
$2x=r+1$; $y=r-3$; $4-r=z$
$x=\frac{r+1}{2}$
$(x,y,z)=\left(\frac{r+1}{2}, r-3, 4-r\right)$ lies
on sphere ⇒ x,y,z lie on eqn ②

$\left(\frac{r+1}{2}\right)^2+(r-3)^2+(4-r)^2-\overset{3}{\cancel{6}}\left(\frac{r+1}{\cancel{2}}\right)+8(r-3)-4(4-r)+4$

$\frac{(r+1)^2}{2^2}+r^2+9-6r+16+r^2-8r-3r-3+8r-24-16+\ldots+4=$

$\frac{r^2+1+2r}{4}+2r^2-5r-14=0$
$\frac{r^2+1+2r+8r^2-20r-56}{4}=0 \Rightarrow 9r^2-18r-55=0$

18×18
324

36×5
180
180
1980

$9r^2-18r-55=0$.

is a Quadratic eqn of $ax^2+bx+c=0$

$\Rightarrow$ we know $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$.

here: $a=9$ $b=-18$, $c=-55$

$$r=\frac{-(-18)\pm\sqrt{(-18)^2-4(9)(-55)}}{2(9)}=\frac{18\pm\sqrt{324+19}}{18}$$

$$r=\frac{18\pm\sqrt{2304}}{18} \quad r=\frac{18\pm48}{18} \Rightarrow r=\frac{18+48}{18}, \frac{18-48}{18}=\frac{30}{18}$$

$r=\frac{11}{3}$; $r=\frac{-5}{3}$

if $r=\frac{11}{3}, \frac{-5}{3}$

$$(x,y,z)=\left(\frac{r+1}{2}, r-3, -r+4\right)$$

if $r=\frac{11}{3}$ if $r=\frac{-5}{3}$

$$(x,y,z)=\left(\frac{\frac{11}{3}+1}{2}, \frac{11}{3}-3, 4-\frac{11}{3}\right) \& \left(\frac{\frac{-5}{3}+1}{2}, \frac{-5}{3}-3, \frac{4+5}{3}\right)$$

$\left(\frac{\frac{11+3}{3}}{2}\right)$

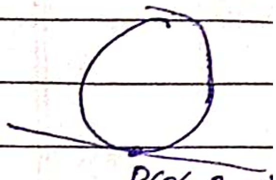
$$(x,y,z)=\left(\frac{11+3}{3\times2}, \frac{11-9}{3}, \frac{12-11}{3}\right) \& \left(\frac{-5+3}{3\times2}, \frac{-5-9}{3}, \frac{12+5}{3}\right)$$

$$(x,y,z)=\left(\frac{14}{6}, \frac{+2}{3}, \frac{1}{3}\right) \& \left(\frac{-2}{6}, \frac{-14}{3}, \frac{17}{3}\right)$$

point of intersect are

$$\left(\frac{7}{3}, \frac{2}{3}, \frac{1}{3}\right) \& \left(\frac{-1}{3}, \frac{-14}{3}, \frac{17}{3}\right)$$

⑫ If tangent plane to sphere $x^2+y^2+z^2=r^2$ makes intercepts a, b, c on co-ordinate axes ⇒ ST $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{r^2}$



given eqn of sphere is

$x^2+y^2+z^2=r^2$ —①

Here, centre $(-u,-v,-w)=(0,0,0)$.

let consider a point $p(\alpha,\beta,\gamma)$

⇒ Tangent plane to —① at p is

$(\alpha+u)x+(\beta+v)y+(\gamma+w)z+\overset{u^2}{\cancel{0}}+\overset{v^2}{0}+\overset{+w^2}{\cancel{0}}-d=$

$u\alpha+v\beta+w\gamma-$

~~$\alpha x+\beta$~~

$(\alpha+0)x+(\beta+0)y+(\gamma+0)z+0+0+0-r^2=0$

$\alpha x+\beta y+\gamma z-r^2=0$ —②

Now if This plane makes intercepts a, b, c on axes ⇒.

~~$x=a,\ y=b,\ z=c \Rightarrow \alpha(a)+\beta(y)+\gamma(c)-r^2=0.$~~

a

3 points $A(a,0,0)$ $B(0,b,0)$ $C(0,0,c)$ possible

Sub A in eqn ② $\alpha(a)+0+0-r^2=0 \Rightarrow \alpha=\frac{r^2}{a}$

" B in eqn ② $0+\beta(b)+\gamma(0)\overset{-r^2}{}=0 \Rightarrow \beta=\frac{r^2}{b}$

" C in eqn ② $\Rightarrow \alpha(0)+\beta(0)+\gamma(c)-r^2=0 \Rightarrow \gamma=\frac{r}{c}$

~~Sub~~ (α,β,γ) is point lies on sphere

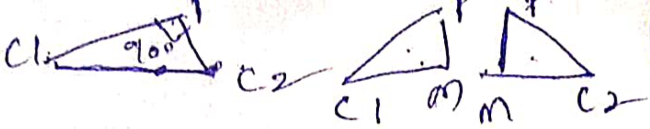
⇒ sub α,β,γ in eqn ①.

$x^2+y^2+z^2=r^2$

$\left(\frac{r^2}{a}\right)^2+\left(\frac{r^2}{b}\right)^2+\left(\frac{r^2}{c}\right)^2=r^2$

$(r^2)^{\cancel{2}}\left[\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right]=r^{\cancel{2}} \Rightarrow \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{r^2}$

Hence proved.



radius of common circle is PM

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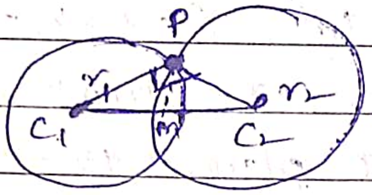
(14) TO spheres of radii r_1 & r_2 cut orthogonally ⇒ PT radius of common circle $\frac{r_1 r_2}{\sqrt{r_1^2+r_2^2}}$ ✓

let us consider 2 ~~circles~~ spheres as shown bellow of raddi r_1 & r_2 & centres C_1, C_2.



PM is radius for common circle

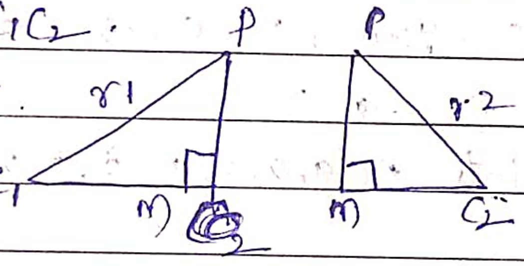
⇒ if They cut orthogonally ⇒ $\angle C_1 P C_2 = 90°$

& we know

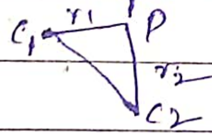
$C_1P = r_1$ $C_2P = r_2$

let draw a perpendicular from P onto C_1C_2.

⇒ we get



as 2 right angle Triangles and The

initial  is a right angle from begining.

⇒ we can apply pythogoreous Theorem to all Thes 3 △les.

for ΔC_1MP

for ΔPMC_2

$r_1^2 = C_1M^2 + PM^2$

$C_1M^2 = r_1^2 - PM^2$

$C_1M = \sqrt{r_1^2 - PM^2}$

$PM^2 + MC_2^2 = r_2^2$

$C_2M^2 = r_2^2 - PM^2$

$C_2M = \sqrt{r_2^2 - PM^2}$

$(a+b)^2 = a^2+b^2+2ab$.

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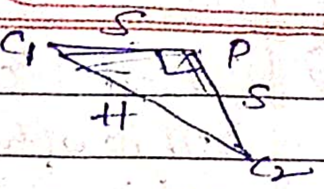
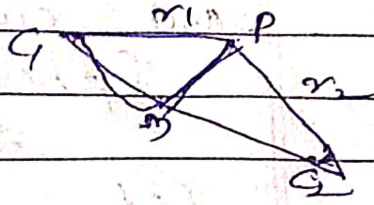
for 1st ΔC_1PC_2.



$H^2 = S^2+S^2$



$~~C_1C_2~~\,(C_1C_2)^2 = r_1^2+r_2^2$

$(C_1m+C_2m)^2 = r_1^2+r_2^2$ ✓

$$\left[\sqrt{r_1^2-pm^2}+\sqrt{r_2^2-pm^2}\right]^2 = r_1^2+r_2^2$$

$$\left(\sqrt{r_1^2-pm^2}\right)^2+\left(\sqrt{r_2^2-pm^2}\right)^2+2\sqrt{r_1^2-pm^2}\sqrt{r_2^2-pm^2} = r_1^2+r_2^2$$

$$\cancel{r_1^2}-pm^2+\cancel{r_2^2}-pm^2+2\sqrt{r_1^2-pm^2}\sqrt{r_2^2-pm^2} = \cancel{r_1^2}+\cancel{r_2^2}$$

$$2\sqrt{r_1^2-pm^2}\sqrt{r_2^2-pm^2} = 2pm^2$$

SBS.

$$(r_1^2-pm^2)(r_2^2-pm^2) = (pm^2)^2$$

$$r_1^2r_2^2 - r_1^2pm^2 - r_2^2pm^2+(pm^2)^2 = pm4$$ ✓

$$r_1^2r_2^2 = \cancel{pm4}-\cancel{pm4}+r_1^2pm^2+r_2^2pm^2$$

$$r_1^2r_2^2 = (r_1^2+r_2^2)pm^2$$ ✓

$$pm^2 = \frac{r_1^2r_2^2}{r_1^2+r_2^2}$$

$$pm = \sqrt{\frac{r_1^2r_2^2}{r_1^2+r_2^2}} = \frac{r_1r_2}{\sqrt{r_1^2+r_2^2}}$$

SO radius of compn circle pm is

$\dfrac{r_1r_2}{\sqrt{r_1^2+r_2^2}}$ Hence proved.

⑮ find equation of sphere that passes Through points $(0,3,0)$ $(-2,-1,-4)$ and cuts orthogonally 2 Spheres $x^2+y^2+z^2+x-3z-2=0$

$2(x^2+y^2+z^2)+x+3y+4=0$

let eqn of sphere be

$x^2+y^2+z^2+2ux+2vy+2wz+d=0$ — ①

eqn ① passes through $(0,3,0)$

$0^2+3^2+0^2+2u(0)+2v(3)+2w(0)+d=0$

$9+6v+d=0 \Rightarrow d=-9-6v$ — ②

$\Rightarrow 6v+9+d=0$

eqn ① pass Through $(-2,-1,-4)$

$(-2)^2+(-1)^2+(-4)^2+2u(-2)+2v(-1)+2w(-4)+d=0$

$4+1+16-4u-2v-8w+d=0$

$4u+2v+8w-d-21=0$ — ③

given That eqn ① cuts orthgonaly 2spheres $x^2+y^2+z^2+x-3z-2=0$

$\Rightarrow 2u_1=1$; $2v_1=0$, $2w_1=-3$ $d_1=-2$

$u_1=\frac{1}{2}$, $v_1=0$, $w_1=\frac{-3}{2}$

If it cuts ① orthogonally $\Rightarrow$

$2u_1u_2+2v_1v_2+2w_1w_2=d_1+d_2$

$2u(\frac{1}{2})+2v(0)+2w(\frac{-3}{2})=d-2$

$u-3w-d+2=0$ — ④

2nd sphere is $2(x^2+y^2+z^2)+x+3y+4=0$

$x^2+y^2+z^2+\frac{x}{2}+\frac{3y}{2}+2=0$

$2u_2=\frac{1}{2}$ $\Rightarrow u_2=\frac{1}{4}$ $2v_2=\frac{3}{2}$ $v_2=\frac{3}{4}$ $w_2=0$ $d_2=2$

[illegible] solving [illegible] -1 ought [illegible]
So mee ku [illegible] solve [illegible]

from condition of orthogonality

$2u_1u_2+2v_1v_2+2w_1w_2=d_1+d_2$

$2u(\frac{1}{4})+2v(\frac{3}{4})+2w(0)=d+2$

$\frac{u}{2}+\frac{3v}{2}=d+2$

$u+3v-2d-4=0$ — ⑤

we have ④ eqns ② ③ ④ ⑤ if we solve them $\Rightarrow$ we get $u,v,w,d \Rightarrow$ sphere eqn.

② + ③	② + ④	②×2 + ⑤
$6v+d+9=0$	$6v+d+9=0$	$12v+18+2d=0$
$4u+2v-d+8w-21=0$	$u-3w-d+2=0$	$u+3v-4-2d=0$
$4u+8v+8w-12=0$	$u+6v-3w+11=0$	$u+15v+14=0$
I	II	III

Solve.

II − III

$u+6v-3w+11=0$

$u+15v+14=0$

$-9v-3w-3=0$

$-3[3v+w+1]=0$

$-3\neq0 \Rightarrow 3v+w+1=0$ — ⑥

II×4 − I

$4u+24v-12w+44=0$

$4u+8v+8w-12=0$

$+16v-20w+56=0$

$4[4v-5w+14]=0$

$4v-5w+14=0$ ⑦

add both

$3v+w+1=0$

$4v+w+14=0$

$7v+15=0 \Rightarrow$

add solve ⑥ $\overset{5}{\times}$ ⑤ & ⑦.

$$15v+5w+5=0$$
$$4v\mp 5w+14=0$$
$$\text{(11)}\quad 19v+19=0 \Rightarrow 19v=-19$$
$$\boxed{v=-1}$$

sub v in ⑦ ⇒

$$4v-5w+14=0$$
$$4(-1)-5w+14=0$$
$$-4-5w+14=0$$
$$10=5w \Rightarrow \boxed{w=2}$$

sub v in eqn ②

$$6v+d+9=0$$
$$6(-1)+d+9=0 \Rightarrow d=6+9=0$$
$$\boxed{d=-3}$$

sub w, v, d in $u+3v-2d-4=0$

$$u+3(-1)-2(-3)-4=0$$
$$u-3+6-4=0$$
$$u-7+6=0 \Rightarrow u=1$$

∴ $u=1$, $v=-1$, $w=2$ & $d=-3$

Sub in eqn ①

$$x^2+y^2+z^2+2(1)x+2(-1)y+2(2)z-3=0$$
$$x^2+y^2+z^2+2x-2y+4z-3=0$$

∴ $x^2+y^2+z^2+2x-2y+4z-3=0$ Req

eqn of sphere

① Find limiting points of co-axial System
$x^2+y^2+z^2-20x+30y-~~40z~~-40z+29~~+0~~$
$+\lambda(2x-3y+4z)=0$

Soln :— given co-axial system eqution is

$$x^2+y^2+z^2-20x+30y-40z+29+\lambda(2x-3y+4z)=0$$

comparing with general eqution → ~~[illegible]~~ we get
$x^2+y^2+z^2+(-20+2\lambda)x+(30-3\lambda)y+(-40+4\lambda)z+29=0$

$2u=(-20+2\lambda)$ and $2v=30-3\lambda$, $2w=-40+4\lambda$ $d=29$

$u=-10+\lambda$ $v=15-\frac{3}{2}\lambda$ $w=-20+2\lambda$

as we know radius of Co-axial system is zero

$r=\sqrt{u^2+v^2+w^2-d}=0$

$r\Rightarrow\sqrt{(-10+\lambda)^2+(15-\frac{3}{2}\lambda)^2+(-20+2\lambda)^2-29}=0$

$\lambda^2+100-20\lambda+225+\frac{9}{4}\lambda^2-45\lambda+4\lambda^2+400-80\lambda-29=0$

$5\lambda^2+\frac{9}{4}\lambda^2-145\lambda+696=0$

$20\lambda^2+9\lambda^2-580\lambda+2784=0$

$29\lambda^2-580\lambda+2784=0$

$29(\lambda^2-20\lambda+96)=0$

$\lambda^2-20\lambda+96=0$

$\lambda^2-12\lambda-8\lambda+96=0$

$\lambda(\lambda-12)-8(\lambda-12)=0$

$(\lambda-12)(\lambda-8)=0$

$\lambda=12, 8$

1 2
145×4
580

2784/29

8 5
29×96
864
192
~~[illegible]~~
2784

96
12 8

we have $u = -10 + \gamma$ $\quad v = 15 - \frac{3}{2}\gamma$ $\quad w = -20 + 2\gamma$

when $\gamma = 12 \Rightarrow u = -10 + 12$ $\quad v = 15 - \frac{3}{2} \cdot 12$ $\quad w = -20 + 2(12)$

$u = 2$ $\quad v = -3$ $\quad w = 4$

$\gamma = 8 \Rightarrow u = -10 + 8$ $\quad v = 15 - \frac{3}{2}(8)$ $\quad w = -20 + 2(8)$

$u = -2$ $\quad v = 3$ $\quad w = -4$

$\Rightarrow$ limiting points $(-u, -v, -w)$ is

$(-2, 3, -4)$

$(2, -3, 4)$

find the limiting points of the coaxial system defin

efined by spheres $x^2+y^2+z^2+4x-2y+2z+6=0$ and $x^2+y^2+z^2$

$x^2+y^2+z^2+2x-4y-2z+6=0$.

Answer:-

Given spheres are,

$S_1: x^2+y^2+z^2+4x-2y+2z+6=0$

$S_2: x^2+y^2+z^2+2x-4y-2z+6=0$

The equation of plane of circle through two spheres is,

$S_1-S_2=0$

$\Rightarrow x^2+y^2+z^2+4x-2y+2z\ 6-(x^2+y^2+z^2+2x-4y+2z+6)=0$

$\Rightarrow x^2+y^2+z^2+4x-2y+2z+6-x^2-y^2-z^2-2x+4y+2z-6=0$

$\Rightarrow 2x+2y+4z=0$

$\Rightarrow x+y+2z=0$

Equation of co-axial system of spheres is $\boxed{S_1+\lambda(S_1-S_2)=}$

$x^2+y^2+z^2+4x-2y+2z+6+\lambda(x+y+2z)=0$

ie, $x^2+y^2+z^2+4x-2y+2z+6+lx+ly+2lz=0$

$\Rightarrow x^2+y^2+z^2+4x-2y+2z+6+lx+ly+2lz=0$

$\Rightarrow x^2+y^2+z^2+x(4+\lambda)+y(-2+\lambda)+2(1+\lambda)z+6=0$.

comparing above equatioon with standard equation of sphere

$x^2+y^2+z^2+2ux+2+2vy+2wz+d=0$.

$u=\frac{4+l}{2}$ $v=\frac{-2+l}{2}$ $w=1+l$ $d=6$

$\therefore$ centre $=\left[-\left(\frac{4+l}{2}\right), -\left[\frac{-2+l}{2}\right], -(1+l)\right]$

Radius, $r=\sqrt{\left[-\left[\frac{4+l}{2}\right]\right]^2+\left[-\left[\frac{-2+l}{2}\right]\right]^2+\left(-(1+l)\right)^2-6}-$

$=\sqrt{\frac{(4+l)^2}{4}\ \frac{(-2+l^2)^2}{4}+(1+l)^2 6}=\sqrt{\frac{16+8l+l^2}{4}+\frac{l^2+4-4l}{4}+l^2+2}$

$$= \sqrt{\frac{16+8l+l^2+l^2+4-4l+4l^2+8l+4-24}{4}}$$

$$\Rightarrow r = \sqrt{\frac{6l^2+12l}{4}} = \sqrt{\frac{6\lambda^2+12\lambda}{4}}$$

for limiting points of co-axial system radius is zer

ie, $\sqrt{\frac{6l^2+12l}{4}} = 0$

$$\Rightarrow 6l^2+12l=0 \qquad \boxed{l=\lambda}$$

$$\Rightarrow 6l(l+2)=0$$

$$\Rightarrow 6l=0,\ l+2=0$$

$$\Rightarrow l=0,\ l=-2$$

for $l=0$

$$\text{centre} = \left[-\left[\frac{4+0}{2}\right], -\left[\frac{-2+0}{2}\right], -(1+0)\right] = \left[\frac{-4}{2}, -(-1), -1\right]$$

$$= (-2, 1, -1)$$

for $l=-2$,

$$\text{centre} = \left[-\left[\frac{4-2}{2}\right] - \left[\frac{-2-2}{2}\right] - (1,-2)\right] = \left[\frac{-2}{2}, -(-2) -(-1)\right.$$

$$= -(1, 2, 1)$$

∴ The limiting points are $(-2, 1, -1)$ and $(-1, 2, 1]$.

55) find the limiting points of the co-axial system defined by the spheres. $x^2+y^2+z^2+3x-3y+6=0$ and $x^2+y^2+z^2-6y-6z+6=0$.

Answer:- Given spheres are.

$S_1 : x^2+y^2+z^2+3x-3y+6=0$

$S_2 : x^2+y^2+z^2-6y-6z+6=0$

The equation of plane of circle through two spheres is,

$$S_1-S_2=0$$

$$\Rightarrow x^2+y^2+z^2+3x\overset{S_1}{\downarrow}-3y+6-x^2\overset{-S_2}{\downarrow}-y^2-z^2+6y+6z-6=0$$

$$\Rightarrow 3x+3y+6z=0$$

$$\Rightarrow 3[x+y+2z]=0$$

Equation of co-axial system of is $S_1+l(x+y+2z)=0$

ie, $x^2+y^2+z^2+3x-3y+6+l(x+y+2z)=0$

$\Rightarrow x^2+y^2+z^2+3x-3y+6+lx+ly+2lz=0$

$\Rightarrow x^2+y^2+z^2+(3+l)x+(-3+l)y+2lz+6=0$

comparing equation (1) with $x^2+y^2+z^2+2ux+2vy+2wz+d=0$

$$u=\frac{3+l}{2}, v=\frac{l-3}{2}; w=l; d=6$$

$\therefore$ centre $= \left[-\left[\frac{3+l}{2}\right], -\left[\frac{l-3}{2}\right], -l\right]$

$(-u,-v,-w)$

Radius $r = \left[\left[\frac{3+l}{2}\right]+\left[\frac{l-3}{2}\right]^2+l^2-6\right]^{\frac{1}{2}} = \left[\frac{(3+l)^2}{4}+\frac{(l-3)^2}{4}+l^2-6\right]^{\frac{1}{2}}$

$= (u^2+v^2+w^2-r^2)^{1/2}$

$$=\left[\frac{9+l^2+6l+l^2+9-6l+4l^2-24}{4}\right]^{\frac{1}{2}}$$

$$=\left[\frac{6l^2-6}{4}\right]^{\frac{1}{2}}$$

$$\Rightarrow r=\frac{1}{2}\left[6l^2-6\right]^{\frac{1}{2}}$$

Squaring and equating radius to zero, we get limiting points here

$$^2=0 \Rightarrow 6l^2-6=0$$

$$\Rightarrow 6l^2=6$$

$$\Rightarrow l^2=1$$

$$\therefore l=\pm 1$$

for $l=1$.

$$\text{centre}=\left[-\left(\frac{3+1}{2}\right),-\left(\frac{1-3}{2}\right),-(1)\right]$$

$$=(-2,1,-1)$$

for $l=-1$

$$\text{centre}=\left[-\left(\frac{3+1}{2}\right),-\left(\frac{-1-3}{2}\right),-(-1)\right]$$

$$=(-1,2,1)$$

$\therefore$ The limiting points are

$(-2,1-1),(-1,2,1)$

Q58. Find the equations to the two spheres of the co-axal system $x^2 + y^2 + z^2 - 5 + \lambda(2x + y + 3z - 3) = 0$ which touch the plane 3x + 4y = 15.

(OU) June/July-19, Q9(b)

Answer :

Given,

Equation of co-axal system is,

easy

$$x^2 + y^2 + z^2 - 5 + \lambda(2x + y + 3z - 3) = 0$$

$$\Rightarrow \quad x^2 + y^2 + z^2 - 5 + 2\lambda x + \lambda y + 3\lambda z - 3\lambda = 0$$

Do it

$$\Rightarrow \quad x^2 + y^2 + z^2 + 2\lambda x + \lambda y + 3\lambda z - (5 + 3\lambda) = 0 \quad \dots (1)$$

Comparing equation (1) with $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz - d = 0$,

$$2u = 2\lambda \; ; \quad 2v = \lambda \quad ; \quad 2w = 3\lambda \; ; \; d = -(5 + 3\lambda)$$

$$\Rightarrow \quad u = \lambda \; ; \; v = \frac{\lambda}{2} \quad ; \quad w = \frac{3\lambda}{2}$$

$$\text{Centre} = \left(-\lambda, -\frac{\lambda}{2}, -\frac{3\lambda}{2}\right)$$

$$\text{Radius, } r = \sqrt{(\lambda)^2 + \left(\frac{\lambda}{2}\right)^2 + \left(\frac{3\lambda}{2}\right)^2 + 5 + 3\lambda}$$

$$= \sqrt{\lambda^2 + \frac{\lambda^2}{4} + \frac{9\lambda^2}{4} + 5 + 3\lambda}$$

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$$\Rightarrow \quad r = \sqrt{\frac{7}{2}\lambda^2 + 3\lambda + 5}$$

Since equation (1) touches the plane $3x + 4y - 15 = 0$,

Then the perpendicular distance from centre of equation (1) is equal to radius.

$$\text{i.e.,} \quad \frac{\left|3(-\lambda) + 4\left(-\frac{\lambda}{2}\right) - 15\right|}{\sqrt{(3)^2 + (4)^2}} = \sqrt{\frac{7}{2}\lambda^2 + 3\lambda + 5}$$

$$\Rightarrow \quad \frac{|-3\lambda - 2\lambda - 15|}{5} = \sqrt{\frac{7}{2}\lambda^2 + 3\lambda + 5}$$

$$\Rightarrow \quad |-(5\lambda + 15)| = 5\sqrt{\frac{7}{2}\lambda^2 + 3\lambda + 5}$$

Squaring on both sides,

$$(5\lambda + 15)^2 = 25\left(\frac{7}{2}\lambda^2 + 3\lambda + 5\right)$$

$$\Rightarrow \quad 25\lambda^2 + 225 + 150\lambda = \frac{175}{2}\lambda^2 + 75\lambda + 125$$

$$\Rightarrow \quad \frac{175}{2}\lambda^2 + 75\lambda + 125 - 25\lambda^2 - 225 - 150\lambda = 0$$

$$\Rightarrow \quad \frac{125}{2}\lambda^2 - 75\lambda - 100 = 0$$

$$\Rightarrow \quad 125\lambda^2 - 150\lambda - 200 = 0$$

$$\Rightarrow \quad 5\lambda^2 - 6\lambda - 8 = 0$$

$$\Rightarrow \quad 5\lambda^2 - 10\lambda + 4\lambda - 8 = 0$$

$$\Rightarrow \quad 5\lambda(\lambda - 2) + 4(\lambda - 2) = 0$$

$$\Rightarrow \quad (\lambda - 2)\,(5\lambda + 4) = 0$$

$$\Rightarrow \quad \lambda = 2, \lambda = -\frac{4}{5}$$

Substituting the values of λ in equation (1),

For $\lambda = 2$,

$$x^2 + y^2 + z^2 + 2(2)x + 2y + 3(2)z - (5 + 3(2)) = 0$$

$$\Rightarrow \quad x^2 + y^2 + z^2 + 4x + 2y + 6z - 11 = 0$$

For $\lambda = -\frac{4}{5}$,

$$x^2 + y^2 + z^2 + 2\left(-\frac{4}{5}\right)x + \left(-\frac{4}{5}\right)y + 3\left(-\frac{4}{5}\right)$$

$$z - \left[5 + 3\left(-\frac{4}{5}\right)\right] = 0$$

$$\Rightarrow \quad x^2 + y^2 + z^2 - \frac{8}{5}x - \frac{4}{5}y - \frac{12}{5}z - \frac{13}{5} = 0$$

$$\Rightarrow \quad 5(x^2 + y^2 + z^2) - 8x - 4y - 12z - 13 = 0$$

The required equations of spheres are,

$x^2 + y^2 + z^2 + 4x + 2y + 6z - 11 = 0$

$5(x^2 + y^2 + z^2) - 8x - 4y - 12z - 13 = 0.$

Formulas

formulas unit-1

① → Sphere eqn $x^2+y^2+2ux+2vy+2wz+d=0$

centre $(-u, -v, -w)$

radius $r=\sqrt{u^2+v^2+w^2-d}$

② → when a circle eqn & plane eqn is given ⇒ circle eqn + K(plane) = 0 is require Sphere

K can be found by Substitu given point in Sphere eqn

③ → great circle, circle given ⇒ circle + K(great circle) = 0 is sphere and centre of Sphere lies on great circle so by sub $(-u, -v, -w)$ in great circle u will find value of K.

④ → $x^2+y^2+z^2+2u_1x+2v_1y+2w_1z+d_1=0$

$x^2+y^2+z^2+2u_2x+2v_2y+2w_2z+d_2=0$

are eqns of 2 spheres ⇒ if they cut orthogonally ⇒ $2u_1u_2+2v_1v_2+2w_1w_2=d_1+d_2$

⑤ → eqn of Sphere when it ~~passes~~ meets Co-ordinate axes at A, B, C ⇒ A(a,0,0) B(0,b,0) C(0,0,c) ⇒ is

→ $x^2+y^2+z^2-ax-by-cz=0$ [NO d]

eqn of plane Through A, B, C above points

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

⑥ ⇒ sphere eqn passing through origin

$x^2+y^2+z^2+2ux+2vy+2wz=0$

Centere origin ⇒ Sphere $x^2+y^2+z^2=r^2$

⑦ when 2 circles lie on same sphere
That sphere eqn will found by.

1st circle & 1st plane $C_1+KP_1=0$ } ① equate
2nd circle & 2nd plane $C_2+K'P_2=0$ } ② co-effiecient

⇒ find K, K' ⇒ sub in any of ①/② ⇒ i.e. sphe

⑧ eqn of tangent plane through a point (α,β,γ) to a sphere of Centre $(-u,-v,-w)$ is

$u\alpha+v\beta+w\gamma$

$(\alpha+u)x+(\beta+v)y+(\gamma+w)z+\cancel{(u^2+v^2+w^2-d)}$

or $\alpha x+\beta y+\gamma z+(x+\alpha)u+(y+\beta)v+(z+\gamma)w$

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limiting points
↓

ఏ system of radius = 0
దాని యొక్క center is limiting point

⇒ 2 Spheres S_1 & S_2
co-axial system $S_1+\lambda(S_1-S_2)=0$ — ①
లోనున్న దాని
radius = 0 చేస్తే λ
వస్తది once λ వస్తే
eqn ① నుండి $(-u,-v,-w)$
వచ్చే అవే limiting points