UNIT-II

Quantum Mechanics

Dual nature of Matterwaves

Davisson & Germar experiment

gamma ray microscope

photo electric effect

Matterwaves & uncertainty principle

Heisenbergs uncertainity principle

matterwave properties

phase velocity & group velocity expressions.

Sh

properties of wavefunctions

Eigen values & Eigen fuctions

schrodinger wave equation → Time Independent

Time dependent

compton effect

Inadequacy of classical physics

photo electric effect

Einsteen PEE

Inadequate = not enough

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LAQ's

1 (1) wt is compton effect. derive expression for compton shift

(2) Derive schrodinger time Independent eq.n
" " " Dependent eq.n

2 (3) Describe the Experiment of davisson and germer to demonstrate wave nature of electrons.

1 (4) what is photo electric effect Describe its experimental setup

2 (5) what are phase velocity and group velocity and obtain expressions for them

(6) Eigen values and Eigen functions
Expectation values. Explan them

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SAQs

1. Difference b/w phase velocity and group velocity
2. postulates of Quantum mechanics
3. Derive Einsteins photo electric Equation
4. State and Explain Heisenbers uncertainty priniple.
5. physical Significance of wave function
6. properties of Matter waves
7. ~~Heisenb.~~ Gammaray microscope
8. Relation b/w phase velocity and group velocity.

Quantum Mechanics

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3 parts

part - 1. ~~(1)~~ PEE & (2) compton effect ~~(3) Ein~~ LA 3.0m w

~~(3)~~ Eienstein PEE 10m

~~(4)~~ Quantum mechnics postulates 10m

part - 2 :-

w ~~(1)~~ Debroglie wavelength

w (2) Heisenberg uncer priciple

(3) mater waves properties 10m

~~(4)~~ LA phasevelocity & group v differnce 3.8m

w ~~(5)~~ Davison germer experiment

(6) Gamma ray microscope. 10m

Part - 3 :- w 1) Schrong Time d

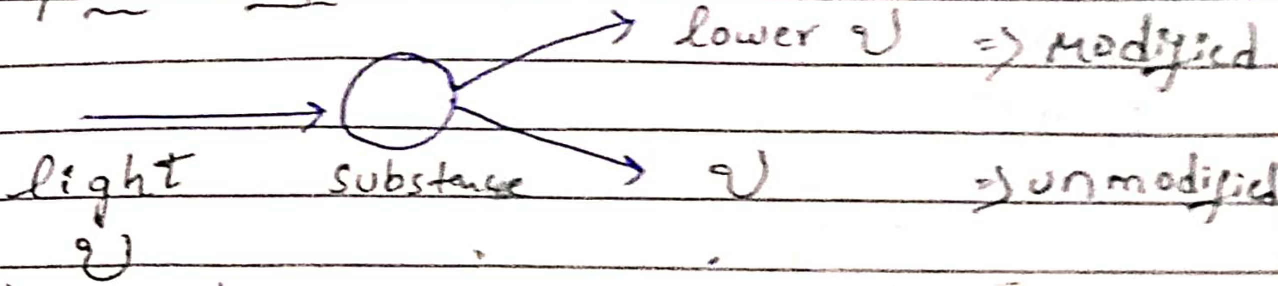
w 2) Time In

2) LA propaties of wave fuction

4) LA eign values eigen fuctions.

1. What is compton Effect? Considering compton effect/scattering derive an expression for shift.

Compton Effect :-



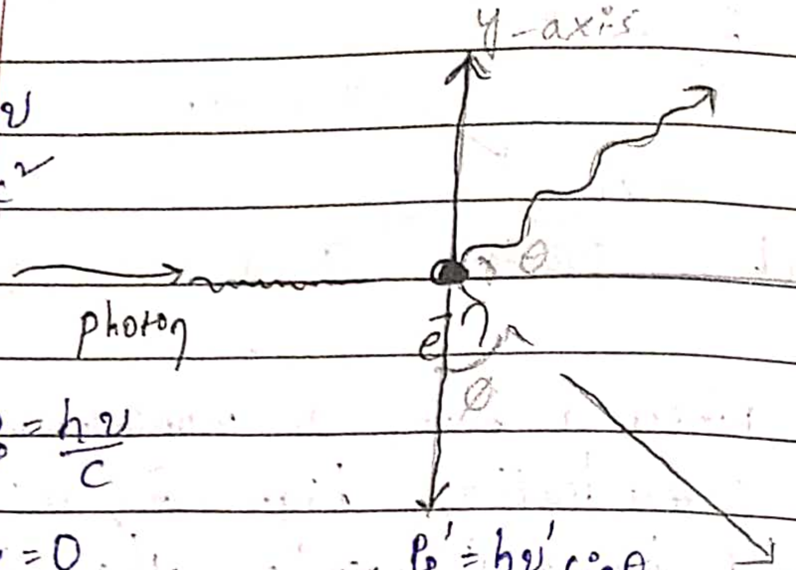
When a beam of monochromatic ~~Source~~ radiation high frequency is Scattered by a Substance, the Scattered radiation consists 2 components

(i) one having lower frequency or greater wavelength which is called as Modified radiation

(ii) The other having Same frequency as original (or) Same wavelength it is called unmodified Radiation.

This phenomenon is called compton effect.

Compton assumed that the e^- is free and at rest before collision with photon. After collision the relative mass of e^- is considered.



$E_p = h\nu$

$E_e = m_0c^2$

$E_p' = h\nu'$

$E_e' = mc^2$

$P_p = \frac{h\nu}{c}$

$P_e = 0$

$P_p' = \frac{h\nu'}{c}\cos\theta$

$P_e' = mv\cos\phi$

$P_p' = \frac{h\nu'}{c}\sin\theta$

$P_e' = mv\sin\phi$

- Let a photon of energy $h\nu$ collides with an e^- at rest.

During collision some part of energy of photon is given to e^- because of which e^- gain KE and recoils.

The process of recoiling of e^- and scattering of photon is shown in fig.

θ is scattering angle of photon & ϕ is recoil angle of e^-.

[recoil = వెళ్లిన వారి వైపే vellaḍam like revenge

$mvc\cos\phi = h\nu - h\nu^1 \cos\theta$

and (4) sobs and adding

$\sin^2\phi + m^2v^2\cos^2\phi = h^2\upsilon^{12}\sin^2\theta + h^2 ...$

$^2(\sin^2\phi + \cos^2\phi) = h^2\upsilon^{12}(\sin^2\theta + \cos^2 ...$

$c^2 = h^2\upsilon^2 + h^2\upsilon^{12} - 2h^2\upsilon\upsilon^1\cos\theta$

$^2c^2 = h^2[\upsilon^2 + \upsilon^{12} - 2\upsilon\upsilon^1\cos\theta]$

of the system Before collision $= h\nu + m_0 ...$

of the system After collision $= h\nu^1 + mc^2 ...$

According to law of conservation of Energy

Case(i) :- when $\theta = 0$ $\cos\theta = 1 \Rightarrow$

$$\Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta) \Rightarrow \Delta\lambda = \frac{h}{m_0 c}(1-1) = 0$$

$\Delta\lambda = 0$ at $\theta = 0$

$\therefore$ No change in frequency/wavenlength of incident radiation.

$\therefore$ No Scattering along direction of incidence.

Case-(ii)

$\theta = \frac{\pi}{2} \Rightarrow \cos\theta = 0$

$$\Delta\lambda = \frac{h}{m_0 c}(1-0) = \frac{h}{m_0 c}$$

$$\Delta\lambda = \frac{6.624 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^{8}}$$

$\Delta\lambda = 0.2426 \times 10^{-10} m$

$\Delta\lambda = 0.02426 A^0$

Compton wave length

case-(iii) $\theta = \pi$

$\cos\theta = -1$

$$\Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

$$= \frac{h}{m_0 c}(1-(-1))$$

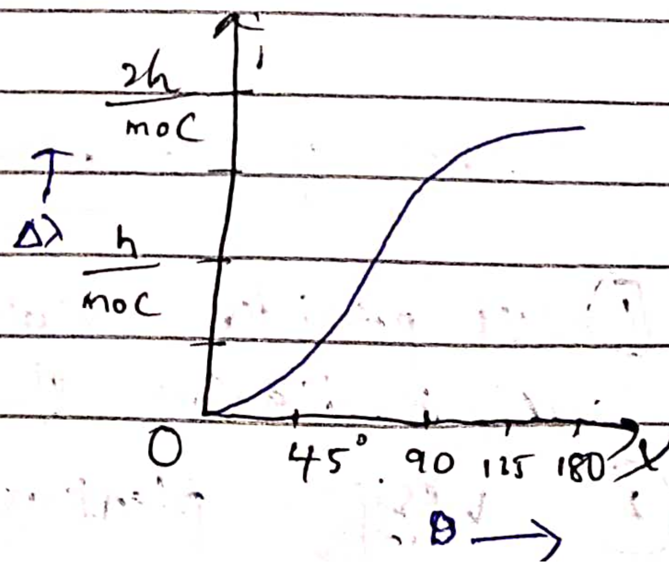
$$\Delta\lambda = 2\frac{h}{m_0 c}$$

$\Delta\lambda = 0.04852 A^0$

when θ change from 0 to 2π

λ also changes.

$\lambda = n\frac{h}{m_0 c}$



frequency of Scattered photon:-

from

$$\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0 c^2}(1-\cos\theta)$$

($\cos\theta = 1 - 2\sin$

$$\frac{1}{\nu'} = \frac{1}{\nu} + \frac{2h}{m_0 c^2} 2\sin^2\frac{\theta}{2}$$

$$\frac{1}{\nu'} = \frac{m_0 c^2 + 2h\nu \sin^2\theta/2}{m_0 c^2 \nu}$$

$$\nu' = \frac{m_0 c^2 \nu}{m_0 c^2 + 2h\nu \sin\theta/2}$$

$$\nu' = \frac{\nu}{1 + \frac{2h\nu}{m_0 c^2}\sin^2\theta/2}$$

$$\boxed{\nu' = \frac{\nu}{1 + 2\alpha \sin^2\theta/2}} \quad \left[\alpha = \frac{h\nu}{m_0 c^2}\right]$$

Uses of compton effect :-

1. It provides evidence of electromagnetic Radiation of particle
2. verifies plankus Quntum hypothesis.

③ verifies relations Indirectly

$$E = mc^2 \quad \text{and} \quad m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

② Derive Einstein's photo electric equation

According Einsteins Explanation in photo electric effect, one photon is completely absorbed by one e^- which there by gains quantum energy and may be emited from metal surface. The photon energy split into 2 parts.

(i) A part of energy is used to seperate $\bar{e}$ from metal surface. This energy is called work function/ W_0

(ii) Another part of energy is used to give KE to $\bar{e}$

$$\boxed{\begin{array}{l} E \text{ of photon} = h\nu \\ h\nu = W_0 + KE \quad -① \end{array}}$$

~~when~~ In some case where photon have an amount of energy value where it can only emit $\bar{e}$ from metal. Then KE of $\bar{e}$ will be zero

$$\frac{1}{2}mv^2 = 0 \Rightarrow \boxed{h\nu_0 = W_0} \quad [\nu = \nu_0]$$

②

Sub (2) in (1).

$$h\nu = h\nu_0 + \frac{1}{2}mv^2$$

$$\boxed{\frac{1}{2}mv^2 = h\nu - h\nu_0 = h(\nu - \nu_0)}$$

from (1) $\frac{1}{2}mv^2 = h\nu - W_0$ - (4) for a parti metal work function W_0 is con

$\Rightarrow \frac{1}{2}mv^2 \propto h\nu \Rightarrow v^2 \propto \nu$

∴ velocity of photo electron is proportional to frequency of Incident Radiation.

If V_0 be stopping potential then

$$\frac{1}{2}mv^2 = eV_0 \quad - (5)$$

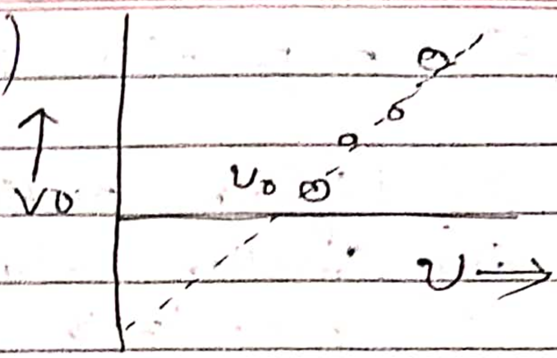
from (3) & (5) $eV_0 = h\nu - h\nu_0$

$$\boxed{V_0 = \frac{h\nu}{e} - \frac{h\nu_0}{e}} \quad - (6)$$

eqn (1) (3) (6) are diffent forms of einsteins PEE.

from ① $V_0 = \left(\frac{h}{e}\right)\nu - \left(\frac{h}{e}\right)\nu_0)$

it represents a straight line.



③ postulates of Quantum Mechanics.

The physics is Study of nature & The study which we did Till 1900's is called classical physics

→ But classical physics had Some rules which failed to prove atomic sub parts in detail

On Dec 1900 max planck gave his presentation where we can study details of atoms on Black body radiation & leading to Quantum mechanics

According to plancks :-

① Emmission of radiation is due to vibration of charged particles in body.

② The radiator body contains SHO of All frequencies

③ The particles contains tiny packet of energies called as photons / Quantum

④ Energy of photon is $E = h\nu$

⑤ general for ossilator $E = nh\nu$ $n = 0, 1, 2, 3 \ldots$

⑥ later Albert Einstien explained PEE

Bohr introduced Bohr atomic model

compton explained compton effect

Based on plancks Theory & Quantum mechanics of planck

(4) properties of matter waves

(i) Lighter particle have greater wavelength

(ii) Smaller velocity particle have greater wavelength

(iii) Velocity = 0 then such particle have infinity wavelength

(v) $v = \infty \Rightarrow$ wavelength = 0 of a particle

$\Rightarrow$ wave is generated by motion of particle

(vi) Velocity of matter wave depends on velocity of matter particle.

(viii) Velocity of matter wave is greater than velocity of light.

(ix) A particle in motion with associated matter waves has two different velocities.

(i) mechanical motion of particle (V) (ii) wave velocity (w)

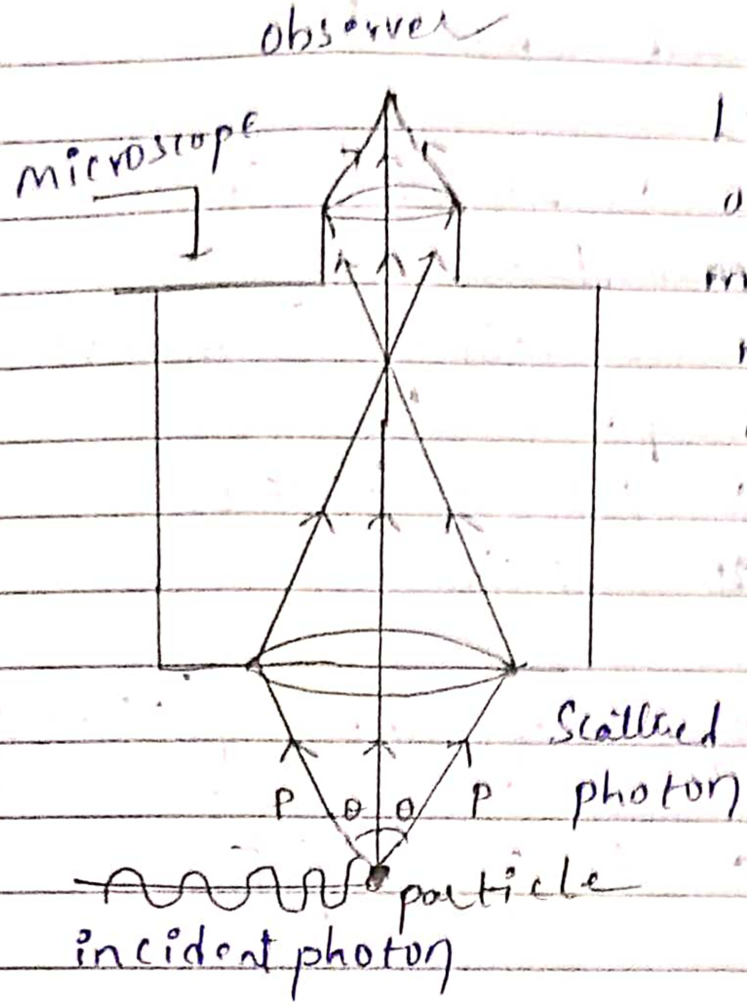
we know

$E = h\nu$ & $E = mc^2 \Rightarrow h\nu = mc^2 \Rightarrow \boxed{\nu = \frac{mc^2}{h}}$

but $w = \nu \times \lambda \Rightarrow w = \frac{mc^2}{h} \times \frac{h}{mV}$

$$\boxed{w = \frac{c^2}{V}}$$

5. write about Gamma ray microscope.



The position and Linear momentum of an e^- can be measured using microscope with an angular apperture 2θ as shown in fig.

Resolving power of microscope is

$$\boxed{\Delta x = \frac{\lambda}{2\sin\theta}} \quad (1)$$

e^- may observed when a photon is Scattered by it into field of view of microscopic lens.

when a photon of Initial momentum $p = \frac{h}{\lambda}$, after Scattering enters the field of view of microscope

It may be anywhere within angle 2θ. Thus, p_x may lie b/w $p\sin\theta$ and $-p\sin\theta$.

According to law of Conservation of momentum,
The uncertainity in momentum is given by.

$$\Delta p_x = p\sin\theta - (-p\sin\theta)$$

$$\Delta p_x = 2p\sin\theta$$

$$\Delta p_x = 2\frac{h}{\lambda}\sin\theta \quad \left[\because \lambda = \frac{h}{p}\right]$$

from 1 & 2 we get

$$\Delta x . \Delta p \simeq \frac{\lambda}{2\sin\theta} \times \frac{2h}{\lambda}\sin\theta$$

$$\boxed{\Delta x . \Delta p_x \simeq h}$$

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(6) what are phase velocity and group velocity obtain the expressions for them

(a) phase velocity / wave velocity (v_p)

The velocity with which planes of constant phase advances through medium is known as phase velocity.

(or)

The ratio of angular frequency and propagation constant is called phase velocity, it denoted by v_p.

Let us consider a wave of single frequency and wavelength, travels through a medium then displacement of wave is given by

$$y = a\sin(\omega t - kx)$$

for planes of constant phase,

$$\omega t - kx = \text{const}$$

on differenciating wrt time,

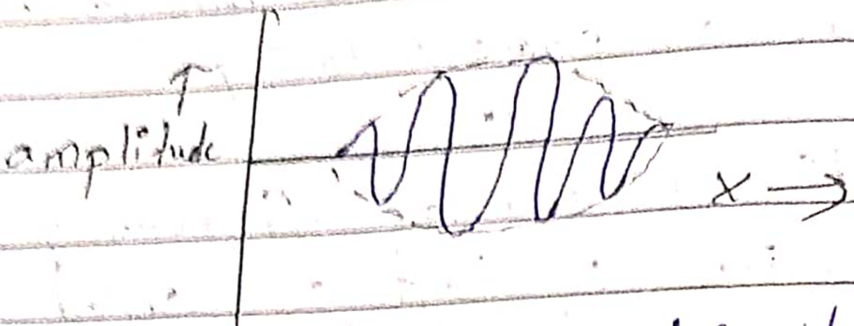
$$\omega \frac{dt}{dt} - k\frac{dx}{dt} = 0$$

$$\omega = k\frac{dx}{dt} \Rightarrow \boxed{\frac{dx}{dt} = \frac{\omega}{k}}$$

$$\boxed{v_p = \frac{\omega}{k}}$$

(ii) Group velocity :- V_g

The velocity with which the energy in group is transmitted is know as group velocity. It is denoted by V_g.



Schrodinger explained dual nature of matter in terms of wave packet

A wave packet contains a group of waves slightly differing in veloc and wavelength, with phases and Amplitudes choosen in such a way that They interfere constructively over a small region of space where the particle can be located and outside this space they interfere destructively so that amplitude reduces to zero. Such a wave packet moves moves with its own veloc called group velocity

The individual waves forming the wave packet have an average velocity called phase velocity.

Consider two waves having same amplitude 'a' slightly different angular frequencies (ω_1 and ω_2) and phase velocities (v_1 and v_2). then the waves can be represented as

$$y_1 = a \sin(\omega_1 t - k_1 x) \quad - (1)$$

$$y_2 = a \sin(\omega_2 t - k_2 x)$$

$\therefore$ According to superpositio principle resultant displacement (y)

$$y = y_1 + y_2$$

$$y = a \sin(\omega_1 t - k_1 x) + a \sin(\omega_2 t - k_2 x)$$

$$y = 2a \cos\left[\left(\frac{\omega_1 - \omega_2}{2}\right) t - \left(\frac{k_1 - k_2}{2}\right) x\right] \sin\left[\left(\frac{\omega_1 + \omega_2}{2}\right) t - \left(\frac{k_1 + k_2}{2}\right) x\right]$$

$$\left(\sin C + \sin D = 2\cos\left(\frac{C-D}{2}\right)\sin\left(\frac{C+D}{2}\right)\right)$$

$$y = 2a \cos\left(\frac{d\omega}{2} t - \frac{dk}{2} x\right) \sin(\omega t - kx)$$

$\therefore$ Amplitude $= 2a \cos\left(\frac{d\omega}{2} t - \frac{dk x}{2}\right)$

Aplitude = $2a \cos\left(\frac{d\omega}{2} t - \frac{dk}{2} x\right)$

Amplitude = $2a \cos \frac{d\omega}{2}\left(t - \frac{dk}{d\omega} x\right)$

Amplitude = $2a \cos \frac{d\omega}{2}\left(t - \frac{x}{V_g}\right)$

$$\boxed{V_g = \frac{d\omega}{dK} = \frac{\omega_1 - \omega_2}{k_1 - k_2}}$$

Relation b/w group velocity and phase velocity :-

we know,

$$V_p = \frac{\omega}{K}$$

$$\Rightarrow \omega = V_p K \Rightarrow d\omega = V_p dK + K dV_p$$

$$\frac{d\omega}{dK} = V_p + K \frac{dV_p}{dK}$$

$$\boxed{V_g = V_p + K \frac{dV_p}{dK}} \quad \text{— (1)}$$

$$\frac{d}{dx}\frac{1}{x} = \frac{-1}{x^2}$$

we know $K = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{K} \Rightarrow d\lambda = \frac{-2\pi}{K^2} dK$

$$\frac{d\lambda}{dK} = \frac{-2\pi}{K^2} \quad \text{— (1)}$$

from (1) $V_g = V_p + K \frac{dV_p}{d\lambda} \frac{d\lambda}{dK}$ — (2)

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from (2) & (3)

$$v_g = v_p + K \cdot \frac{dv_p}{d\lambda}\left(-\frac{2\pi}{K^2}\right)$$

$$\boxed{v_g = v_p - \frac{dv_p}{d\lambda}\frac{2\pi}{K}}$$ $K = 2\frac{\pi}{\lambda}$

$$\boxed{v_g = v_p - \lambda\frac{dv_p}{d\lambda}}$$ ✓ $\boxed{K = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{K}}$

$$\boxed{v_g = v_p + K \frac{dv_p}{dK}}$$

1) write about properties of wave function and physical significance of wave function

→ Wave function is a mathematical function introduced by Schrodinger to explain debroglies ideas of wave.

$$\psi(x,t) = Ae^{-i(kx-\omega t)}$$

properties of wave fuction:-

wave function $\psi(x,y,z,t)$ is the probability that a particle will be found at a place in space at a given time

(i) wave fuction must be finite or zero at any point

(2) wave fuction must have a single valued

(3) wave function be a continuous

(4) wave function posses normalize condition

$$\int_{-\infty}^{\infty} |\psi|^2 \, dx\,dy\,dz = 1$$

→ The probabibility of find system b/w x_1 & x_2 is $\int_{x_1}^{x_2}$ is

$$P = \int_{x_1}^{x_2} |\psi|^2 dx.$$

(b) probability per unit volume is probability density.

$$\Rightarrow \quad \rho = |\psi|^2 dx \;\&$$

$$\int_V \rho \, dV = 1$$

$$\int |\psi|^2 dV = 1$$

physical Significance of wave function

→ By using wave function we can get useful information of system

probability density:-

The wave function is in itself complex $\psi = A + iB$ so its complex conjugate

$$\psi^* = A - iB.$$

$$|\psi|^2 = \psi\psi^*.$$

$$= (A+iB)(A-iB)$$

$$= A^2 + B^2$$ Real ~~Identity~~ quantity

$|\psi|^2$ probability density

The probability of finding a particle at any point at a given time is proportional to probability density

$$|\psi|^2 = \psi\psi^*$$

A large value $|\psi|^2$ means strong possibility of finding particle at the given point of Time.

→ The wave function ψ is different from zero in a finite region which is called wave packet

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(2) Explain Eigen values and Eigen functions also Expectation values.

There is a class of functions ψ which are called Eigen functions.

if for an operator $\hat{O}$ and a constant λ

if $\hat{O}\,\psi(x) = \lambda\,\psi(x)$

then, λ is called eigen value of Operator $\hat{O}$
$\psi(x)$ is eigen function of $\hat{O}$

in Schrodinger wave equation in operator form,
we write,

$$\boxed{\hat{H}\,\psi = E\,\psi}$$

where $\hat{H} = \frac{-\hbar^2}{2m}\nabla^2 + V$ and

$$E = i\hbar\frac{\partial}{\partial t}.$$

So we can define eigen values & eigen functions as

$$\hat{H}\psi = E\psi$$

↓ eigen value

→ eigen function.

The values of Energy/En which can solve schrodinger equation is called eigen value and corresponding function is called eigen ~~value~~ function

Davisson - Germer Experiment

[Experimental Confirmation of matter wave]

→ Davisson & Germer were studying the reflection of electrons from Ni target. The Ni target was subjected to such a heat treatment that the Reflection became Anomalous.

→ The Reflected intensity showed striking maxima & minima.

→ Thus they suspected that the electrons are diffracted like X-rays.

→ The D & G Arrangement shown fig.

Apparatus :-

1) Electron Gun (G)
2) Tungsten filament (F)
3) electron detector (C)
4) Circular Scale (S) → angle [29° to 90°]

The davission and Germer experimental arrangemts is shown in fig The Apperatus consists of a electric gun (G) where the $\bar{e}^{ns}$ are produced and obtained in an electronic beam of known frequency

→ The electric gun consists of tungstan filament (F)

The $\bar{e}^{ns}$ are accelerated in the field.

→ After $\bar{e}^{ns}$ transmited from suitable slits they form a fine beam & directly fall on to Ni, Target

→ The $\bar{e}^{ns}$ acting as waves, diffracted in different directions. The distribution is measured by an $\bar{e}^{n}$ detector which is connected to galvanometer.

→ The $\bar{e}^{n}$ detector [Faraday cylinder (C)] can move on a circular graduated Scale (S) b/w 29° & 90° to recieve reflected $\bar{e}^{ns}$

→ $\bar{e}^{n}$ detector consists of two walls which are insulated from each other

→

A retarding potential is mainted bw them so that [electron detector & walls] only fast moving e^{-ns} coming from gun may enter it.

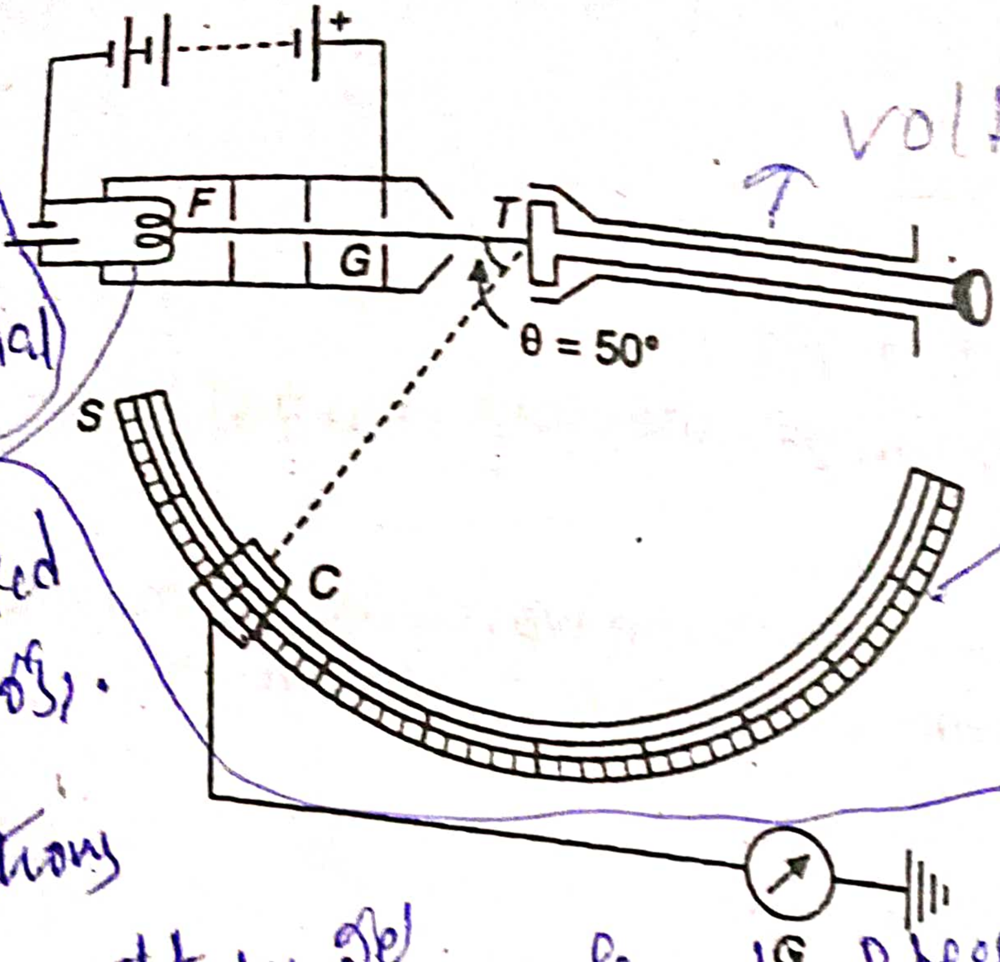
In this way the galvanometer deflection is only due to electrons coming from electron Gun.

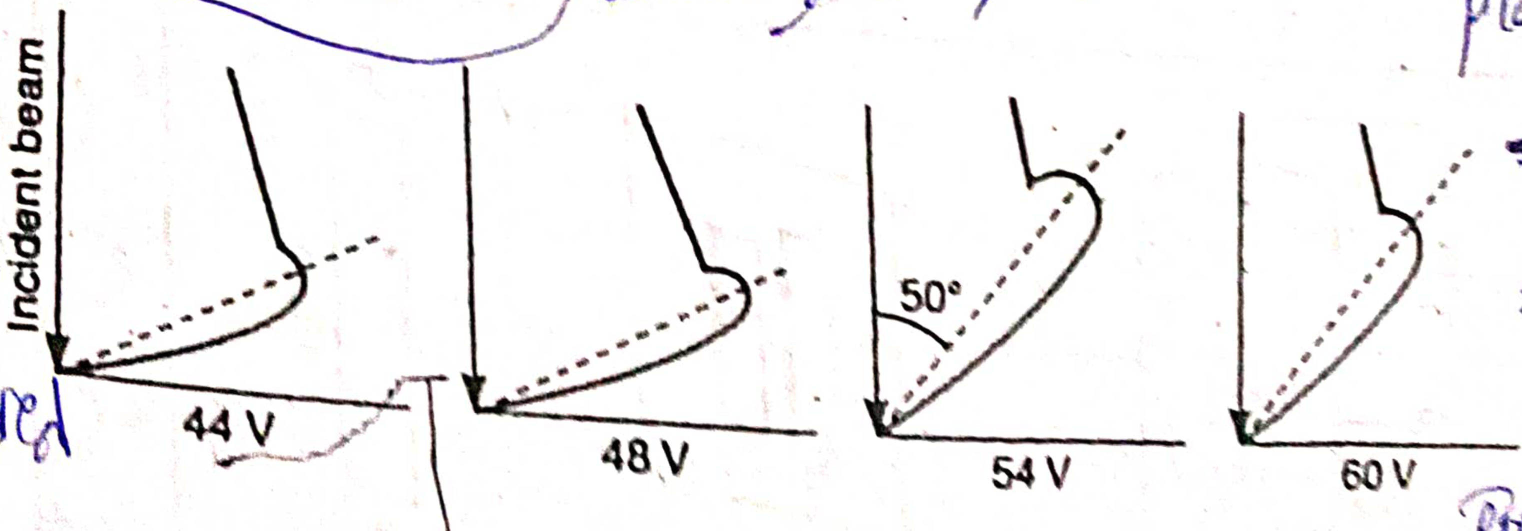
→ The accelarating potential V is given a low value & Crystal is set at any arbitary angle θ.

The electron detector is moved to various positions on Scale 'S' and Galvanometer Current is noted for each position.

Hence it should be remembeed that galvanometer Current is a measurement of intensity of diffracted beam.

A graph is plotted between Galvanometer current and angle ϕ between incident & diffracted beam it comes as shown :





volt

potential

filament → red

-ve electrons

Batteries

fine sharp beam

electron detector

plac

It is observed from graph that are

→ (i) with ↑ potential bump moves ↑ [UW]

(ii) the bump becomes most prominent in curve for (54 V) $\bar{e}^{ns}$ at 50°

(iii) At higher potentials, bump gradually disappears.

⇒ According to De-Broglie, wavelength associated with $\bar{e}^{n}$ accelarated through a potential 'V' is given by.

$$\lambda = \frac{12.26}{\sqrt{V}} A^{0}$$

⇒ wavelength associated with 54 V $\bar{e}^{ns}$

$$\lambda = \frac{12.26}{\sqrt{54}} \Rightarrow \boxed{\lambda = 1.67\ A^{0}}$$

In case of x-ray diffraction, the wavelength of x-rays [According to Bragg's eqns] is $\lambda = 1.65\ A^{0}$

This is almost equal to λ computed from De-Broglie hypothesis.

Hence D & G exper confirm Debroglie cmw.

***** 5th**

Photo Electric Effect

Experimentally E

PEE :- "when a beam of light of Suitable frequency (or) wavelength, is incident on a metal Surface Then the electrons are emmitted from that metal Surface"

=> This phenomenon is known as PEE.
The Emmitted electrons are Called as photo Electrons

=> The current created by photo $\bar{e}$? is called photo current

Experimental Study of photoelectric Effect

The Experimental Arrangement. Consists of The following Apparatus.
Mainly 2 parts

1) Quartz Bulb 2) External circuit.

↓

The Small metal plate A and an electrode 'B' is placed inside the evaluated quartz blub.
The plate A is connected to the negative terminal of battery while B is connected to the +ve terminal of battery through Galvanometer G

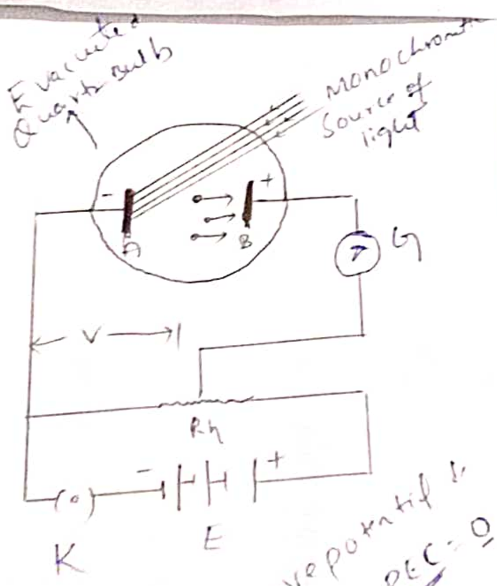
In the absence of light, no electrons flow in the circuit, and hence, there is no deflection in Galvanometer.

But when monochromatic light is allowed to fall on plate A electrons are emmited and Travel towards electrode B and hence Current starts flowing in circuit which is Indicated by Galvanometer. This current is known as photo current.

If the potential of (B) made -ve

=> photo electric current does not Immediatly drop to zero because the photo electrons are emmited from (A) with finite velocity.

-> If -ve potential further increased The photo current decreases and finally Becomes zero at a particular value known as stopping potential, it is denoted by (V_0)



+ve ↑ PEC ↑

-ve potential ↑ PEC = 0

potential ↓ stopping potential.

=> The -ve potential of electrode at which PEC is zero is called. Stopping potential (V_0)

① De-Broglie's hypothesis or [DB λ]

→ According to De broglie hypothesis moving particle is associated with a wave known as De-Broglie wave.

The wavelength of matter wave is given by

$$\boxed{\lambda = \frac{h}{mv} = \frac{h}{p}}$$

5 things to Remember.

① plancks $E = h\nu$

② Eienstien $E = mc^2$

$\left.\right\} \boxed{\lambda = \frac{h}{mc}} \Rightarrow \boxed{c = v}$

③ KE of e^{-n}

④ KE of charge.

⑤ $e^{-ns} \Rightarrow E = KE = \frac{1}{2} m_0 v^2 = eV$

According planck's theory energy of photon,

$$\boxed{E = h\nu} - (1)$$

According to eqn of Eientstiens mass - Energy radiation

$$\boxed{E = mc^2} - (2)$$

from (1) & (2) $h\nu = mc^2$

$$\nu = \frac{mc^2}{h} \Rightarrow \frac{c}{\lambda} = \frac{mc^2}{h}$$

$$\boxed{\lambda = \frac{h}{mc}}$$

In case of material particle with veloci v.

$$\boxed{\lambda = \frac{h}{mv}}$$

(3) KE of particle is $E \Rightarrow E = \frac{1}{2}mv^2$

$$mv^2 = \frac{2E}{m} \Rightarrow m^2v^2 = 2mE$$

$$p^2 = 2mE$$

$$p = \sqrt{2mE}$$

$$\Rightarrow \boxed{\lambda = \frac{h}{\sqrt{2mE}}}.$$

$p \to$ ঊৰ্ধ্ব $\to$ sub
p eleminated
$E \to$ new comer

(4) when charged particle carrying a charge 'q' & potential difference 'V'

$\Rightarrow KE,\ E = qV$ — (6).

from (5) & (6)

$$\boxed{\lambda = \frac{h}{\sqrt{2mqV}}}$$

E → దీన్ని sub.
E → eliminated
q, V → new come

(5) In case of electrons.

$$E = KE = \frac{1}{2} m_0 v^2 = eV$$

$$V = \frac{2eV}{m_0}$$

$$\Rightarrow \lambda = \frac{h}{m_0 V} \Rightarrow \lambda = \frac{h}{m_0}\sqrt{\frac{m_0}{2eV}}$$

$$\lambda = \frac{h}{\sqrt{2eVm_0}}$$

$$\lambda = \frac{12.26}{\sqrt{V}}\ A^0.$$

$\therefore V = 100V \Rightarrow \lambda = \frac{12.26}{\ } A^0 \rightarrow \lambda = 1.226 A^0$

Heisenberg uncertainty principle

It is imposible to specify/measure precisely and simultaneously the values of both members of particular physical variables that describes the behaviour of an atomic System.

(1) position - Momentum uncertainity
(2) Energy - time uncertainity principle

→ The order of magnitude of uncertainities in the knowledge of 2 variables must be atleast planks constent h.

→ Considering the pair of physical variables as 'a' and 'b' then

$$\Delta a \cdot \Delta b \cong h$$

(1) position and momentum uncertainity :-

Δx = uncertainity in mesurement of position
Δp = " " " " momentum

⇒ According to principle

$\Delta x \cdot \Delta p \cong h$ and also → They won't less then order of $\frac{h}{4\pi}$ $\boxed{\Delta x \cdot \Delta p \geq \frac{h}{4\pi}}$

(iii) Energy time uncertainity

ΔE = UIM of Energy
ΔP_x = UIM of momemum
Δt = UIM of time.

$\Rightarrow \Delta E \cdot \Delta t \cong h$ and $\Delta E \cdot \Delta t \geq \frac{h}{4\pi}$.

Proof:- of $\Delta E \Delta t \geq \frac{h}{4\pi}$

Any particle mass m_0 & velocity v_x
$\Rightarrow$ KE

$$E = \frac{1}{2} m_0 v_x^2 = \frac{m_0^2 v_x^2}{2m_0} = \frac{p^2}{2m_0}$$

$$\Delta E = \frac{2 P_x \Delta P_x}{2m_0}.$$

$$\Delta P_x \cdot P_x = m_0 \Delta E$$

$$\Delta P_x = \frac{m_0 \Delta E}{P_x} = \frac{m_0 \Delta E}{m_0 v_x} = \frac{\Delta E}{v_x}$$

$$\boxed{\Delta E = v_x \Delta P_x} \Rightarrow \Delta P_x = \frac{\Delta E}{v_x}.$$

We know $v_x = \frac{x}{t} \Rightarrow v_x \Delta t = \Delta x$.

$$\boxed{\Delta t = \frac{\Delta x}{v_x}} \Rightarrow \Delta x = \Delta t \, v_x$$

$$\Delta E \cdot \Delta t = v_x \Delta P_x \frac{\Delta x}{v_x} = \Delta x \cdot \Delta P_x \cong h$$

$\Rightarrow$ hence $\Delta E \cdot \Delta t \cong h$ & $\boxed{\Delta E \cdot \Delta t \geq \frac{h}{4\pi}}$ from (1)

SCHRODINGER Wave Equation

(1) Time dependent SWE
(2) Time Independent SWE

SW Equations:- To Study the atomic Systems, we need Equation of motion.
(Similarly to Study debroglie's waves associated with particles and describes the motion of particles, we need a fundamental equations in Quantum mechanics known as Schrodinger wave Equations.

Schrodinger presented the ~~wave~~ Wave Equations as a development of De-Broglie ideas of wave properties of matter for this ~~process~~ purpose, Schrodinger introduced a mathematical function (ψ) known as wave function which is the variable quantity associated with the moving particle and is also complex function of the space co-ordinates of the particle and time.

Let, $\psi = Ae^{i(kx-\omega t)}$ — (1)

$$\frac{\partial \psi}{\partial x} = ik A e^{i(kx-\omega t)}$$

$$\frac{\partial^2 \psi}{\partial x^2} = A \cdot i^2 k^2 e^{i(kx-\omega t)}$$

$$\frac{\partial^2 \psi}{\partial x^2} = k^2 \psi \quad \text{— (2)}$$

But we know that $k = \frac{2\pi}{\lambda}$ & $\lambda = \frac{h}{p}$

$$\therefore \quad k = 2\pi \cdot \frac{p}{h}$$

$$k^2 = \frac{4\pi^2 p^2}{h^2}$$

from (2) $\frac{\partial^2 \psi}{\partial x^2} = -\frac{4\pi^2 p^2}{h^2} \psi$

$$p^2 \psi = \frac{-h^2}{4\pi^2} \frac{\partial^2 \psi}{\partial x^2} \quad \text{— (3)}$$

(1) → $\psi = A e^{i(kx-\omega t)}$ — (1')

differentiating (1) w.r to time then

$$\frac{\partial \psi}{\partial t} = +A e^{i(kx-\omega t)} [-i\omega]$$

$$\frac{\partial \psi}{\partial t} = -i\omega\psi \quad \text{— (4)}$$

we know that

$$E = h\nu = \frac{h\omega}{2\pi}$$

$$\omega = \frac{2\pi E}{h}$$

from (4) we get

$$\frac{\partial \psi}{\partial t} = -i \frac{2\pi E}{h} \psi$$

$$\frac{\partial \psi}{\partial t} \cdot \frac{h}{2\pi i} = -E\psi$$

$$E\psi = -\frac{\partial \psi}{\partial t} \; i\hbar \frac{\partial \psi}{\partial t} \quad \left[\because \hbar = \frac{h}{2\pi}\right.$$

$$E\psi = i\hbar \frac{\partial \psi}{\partial t} \quad \text{— (5)}$$

But, we know that

$$E = KE + PE$$

$$E = \frac{p^2}{2m} + V$$

$$E\psi = \frac{p^2}{2m}\psi + V\psi \quad - (6)$$

Sub eqns (3) & (5) in (6)
we get

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{-h^2}{4\pi^2} \frac{\partial^2 \psi}{\partial x^2} \times \frac{1}{2m} + V\psi$$

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi$$

I

$$i\hbar \frac{\partial \psi}{\partial t} = \left[\frac{-\hbar^2}{2m}\Delta^2 + V\right]\psi \rightarrow (7)$$

[In case of 3-Dimensional]

$$\hat{E}\psi = \hat{H}\psi \quad \text{where}$$

$$\hat{E} = i\hbar \frac{\partial}{\partial t} = \text{Energy operator}$$

$$\hat{H} = \frac{-h^2}{2m}\Delta^2 + V = \text{Hamiltonian operator}$$

$$2m \quad \partial x^2$$

Time Independent Schrodinger wave equation

In many situation; PE do not depend on time. In those, The forces that are acting on the PE (V) depend on the position of the particle Under these Conditions $\psi(x,t)$ can be written as the product of a function of x-alone and the other the functions of 't' alone.

Let, $\psi(x,t) = Ae^{i(kx-\omega t)}$

$$\psi(x,t) = Ae^{ikx}e^{-i\omega t}$$

$$\psi(x,t) = \phi(x)\, e^{-i\omega t}$$

$$\frac{\partial \psi}{\partial t} = -i\omega \phi(x)\, e^{-i\omega t}$$

$$\frac{\partial \psi}{\partial t} = -i\omega \psi$$

$$\frac{\partial \psi}{\partial t} = -i.\frac{2\pi E}{h}\psi \quad \left[\omega = \frac{2\pi E}{h}\right.$$

(2)

$$\Rightarrow E\psi = i\frac{h}{2\pi}\frac{\partial\psi}{\partial t}$$

$$E\psi = i\frac{h}{2\pi}\cdot\frac{\partial\psi}{\partial t}$$

$$E\psi = i\hbar\frac{\partial\psi}{\partial t} \Rightarrow \boxed{\hat{E} = i\hbar\frac{\partial}{\partial t}}$$

$$E\psi = \hat{E}\psi$$

on differentiating equation (1) w.r.t x, then

$$\frac{\partial\psi}{\partial x} = e^{-i\omega t}\frac{\partial\phi}{\partial x}$$

$$\frac{\partial^2\psi}{\partial x^2} = e^{-i\omega t}\frac{\partial^2\phi}{\partial x}$$

According to Schrodinger, Time dependent equation,

$$E\psi = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi \quad -(4)$$

Sub (1) $\rightarrow \psi$ & (3) $\rightarrow \frac{\partial^2\psi}{\partial x^2}$ in eqn (4) we get.

$$E.\phi(x)e^{-i\omega t} = -\frac{\hbar^2}{2m}e^{-i\omega t}\frac{\partial^2\phi}{\partial x^2} + V\phi(x)e^{-i\omega t}$$

$$E\phi(x) = -\frac{\hbar^2}{2m}\frac{\partial^2\phi}{\partial x^2} + V\phi(x).$$

$$E\phi(x) - V\phi(x) = -\frac{\hbar^2}{2m}\frac{\partial^2\phi}{\partial x^2}$$

$$\frac{\hbar^2}{2m}\frac{\partial^2\phi}{\partial x^2} + (E-V)\phi(x) = 0$$

$$\frac{\partial^2\phi}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\phi(x) = 0$$

In General, $\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0$.

In Case of 3-Dimensional

$$\boxed{\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0} \quad -(5)$$

for a free particle $V = 0$.

$\therefore$ from (5)

$$\nabla^2\psi + \frac{2mE}{\hbar^2}\psi = 0$$

equ (5).

$$\nabla^2\psi = -\frac{2m}{\hbar^2}(E-V)\psi$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi = E\psi - V\psi$$

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V\right]\psi = E\psi$$

$$\hat{H}\psi = \hat{E}\psi$$

Momentum of System along x-axis

Before collision $= p_p + p_e = \frac{h\nu}{c} + 0$

After collision $= p_p' + p_e' = \frac{h\nu'}{c}\cos\theta + mv\cos\phi$

According to law of conservation of momentum along x-axis

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + mv\cos\phi \quad -(1)$$

$$h\nu = h\nu'\cos\theta + mvc\cos\phi$$

$$mvc\cos\phi = h\nu - h\nu'\cos\theta \quad -(2)$$

$(a-b)^2 = a^2+b^2-2ab$

Momentum of System along y-axis

Before collision = 0

After collision $= p_p' + p_e' = \frac{h\nu'\sin\theta}{c} + mv\sin\phi$

According to law of conservation of momentum along y-axis

$$0 = \frac{h\nu'}{c}\sin\theta + mv\sin\phi$$

$$0 = h\nu'\sin\theta + mvc\sin\phi$$

$$-h\nu'\sin\theta = mvc\sin\phi \quad -(2)$$

$$mvc\sin\phi = -h\nu'\sin\theta \quad -(3)$$

Total Energy of System Before

Before collision $= E_p + E_e = h\nu + m_0c^2$

After collision $= E_p' + E_e' = h\nu' + mc^2$

According to law of conservation of Energy.

$$E_p + E_e = E_p' + E_e'$$

$$h\nu + m_0c^2 = h\nu' + mc^2$$

$$mc^2 = h(\nu - \nu') + m_0c^2 \quad \text{(S.n.s)}$$

$$m^2c^4 = h^2(\nu-\nu')^2 + m_0^2c^4 + 2h(\nu-\nu')m_0c^2$$

$$m^2c^4 = h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu' + m_0^2c^4 + 2h\nu m_0c^2 - 2h\nu' m_0c^2 \quad -(5)$$

$(2)^2 + (3)^2$

LHS^2 + LHS^2 = RHS^2 + RHS^2

$$m^2v^2c^2\cos^2\phi + m^2v^2c^2\sin^2\phi = h^2\nu^2 + h^2\nu'^2\cos^2\theta - 2h\nu h\nu'\cos\theta + h^2\nu'^2\sin^2\theta$$

$$m^2v^2c^2[\cos^2\phi + \sin^2\phi] = h^2\nu^2 - 2h^2\nu\nu'\cos\theta + h^2\nu'^2[\cos^2\theta + \sin^2\theta]$$

$$m^2v^2c^2 = h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu'\cos\theta \quad -(4)$$

eq^n (5) − (4)

$$m^2c^4 - m^2v^2c^2 = h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu' + m_0^2c^4 + 2h\nu m_0c^2 - 2h\nu' m_0c^2 - h^2\nu^2 - h^2\nu'^2 + 2h^2\nu\nu'\cos\theta$$

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$m^2 = \frac{m_0^2}{\left(1-\frac{v^2}{c^2}\right)}(c^2 - v^2)$$

$$m^2 = \frac{m_0^2}{\frac{c^2-v^2}{c^2}} = \frac{m_0^2c^2}{c^2-v^2}$$

$$m^2c^2[c^2 - v^2] = -2h^2\nu\nu'[1-\cos\theta] + 2h(\nu-\nu')m_0c^2 + m_0^2c^4$$

$$\frac{m_0^2c^2}{c^2-v^2}[c^2-v^2] = -2h^2\nu\nu'(1-\cos\theta) + 2h(\nu-\nu')m_0c^2 + m_0^2c^4$$

$$m_0^2c^4 = -2h^2\nu\nu'(1-\cos\theta) + 2h(\nu-\nu')m_0c^2 + m_0^2c^4$$

$$2h^2\nu\nu'(1-\cos\theta) = 2h(\nu-\nu')m_0c^2$$

$$2h(\nu-\nu')m_0c^2 = 2h^2\nu\nu'(1-\cos\theta)$$

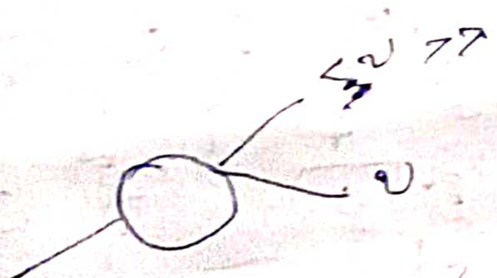
$$\frac{\nu-\nu'}{\nu\nu'} = \frac{h}{m_0c^2}(1-\cos\theta)$$

$$\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0c^2}(1-\cos\theta)$$

$$\frac{c}{\nu'} - \frac{c}{\nu} = \frac{h}{m_0c}(1-\cos\theta)$$

$$\lambda' - \lambda = \Delta\lambda = \frac{h}{m_0c}(1-\cos\theta).$$

$$\Delta\lambda = \frac{h}{m_0c}\left(1-\left(1-2\sin^2\frac{\theta}{2}\right)\right)$$



$$\Delta\lambda = \frac{2h}{m_0c}\sin^2\frac{\theta}{2}$$

λ = change in wavelength

Quantum Mechanics

eg

3 parts

part - 1 : last W ~~(1)~~ PEE & ~~(2)~~ compton effect LA 30m (3) Ein
~~(3)~~ Eienstein PEE 10m
~~(4)~~ Quantum mechnics postulates 10m

part - 2 :-
last W ~~(1)~~ Debroglie wavelength
last W (2) Heisenberg uncer. priciple
~~(3)~~ mater waves properties 10m
~~(4)~~ LA phasevelocity & group v difference 8m
last W ~~(5)~~ Davison germer experiment
(6) Gamma ray microscope. 10m

Part - 3 :- last W 1) Schrodinger Time d
last W 2) Time In
3) LA properties of wave fuction
4) LA eigen values eigen functions.