Maths

1st unit Groups & cyclic groups

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Most important Questions of this chapter for pass Aspirants

5th a (1) Fundamental theorem of cyclic subgroups

(1) State and prove Two step Subgroup Test and ST $H = \{x \in G / x^2 = e\}$ is Subgroup of G when G is abelian

(2) State and prove finite Subgroup Test

(3) Define Centre of group G ST Centre $Z(G)$ is Subgroup of G.

(4) ST every Subgroup of cyclic group is cyclic.

(5) State and prove fundamental theorem of cyclic groups

(6) G is a group and $a \in G$ then $a^i = a^j$ if and only if

(i) a has infinite order then $a^i = a^j$ if and only if $i = j$

(ii) If a has finite order then $a^i = a^j$ if and only if n divides $i - j$

(7) State & prove 2 step Subgroup Test

Remaining Imp Q's of chapter

1. prove the following,
   (i) Intersection of Subgroups is again a Subgroup
   (ii) Identity element is unique in groups
   (iii) Inverse uniqueness " " "
   (iv) Socks-shoes property i.e $(ab)^{-1} = b^{-1}a^{-1}$
   (v) Cancellation laws
   (vi) H and K are 2 Subgroups of G ⇒ ST
   $HK = \{hk \mid h \in H, k \in K\}$ is Subgroup of G
   or product of 2 Subgroups is again a SG
   (vii) every subgroup of cyclic group is abelian

2. a and b are 2 elements a abelian group G if n is any integer ⇒ ST $(ab)^n = a^n b^n$ and $(ab)^{-1} = a^{-1}b^{-1}$.
   is it true for non abelian group?

3. prove that $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is group under matrix multiplication

4. If $*$ defined on Q^+ by $a*b = \frac{ab}{2}$ ⇒ ST $(Q^+, *)$ is a group

5. S is set of real number except -1 define $*$ on S by $a*b = a+b+ab$ ⇒ ST $(S, *)$ is abelian group

6. a be an element of order n in group ⇒ PT, k is +ve num
   $\langle a^k \rangle = \langle a^{\gcd(n,k)} \rangle$ & $|a^k| = \frac{n}{\gcd(n,k)}$ ber

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Unit - 2

Permutation Groups & isomorphisms

Important Questions for pass Aspirants

(8) State and prove Fermat's little theorem

(1) Cayley's theorem

(2) Lagranges theorem

(3) S.T Set of even permutations in S_n forms a subgroup of S_n

(4) State and prove orbit-stabilizer Theorem

Remaining important Questions:-

(1) PT a group of prime order is cyclic

(2) Define cosets and their properties

(3) Find cosets of Subgroup $4Z$ of Z, Z is is group of integers

(4) prove that order of Subgroup H divides order of G and pt $a^{|G|} = e$ (add to Lagranges)

(5) PT for $n > 1$ Alternating group A_n has order $\frac{n!}{2}$

(6) Let $\alpha, \beta \in S_6$ and $\alpha = (1\ 2\ 4\ 5\ 3\ 6)$ $\beta = (1\ 4\ 3\ 2\ 5\ 6)$ ⇒ find $\alpha, \beta, \alpha\beta^{-1}, \alpha^2$

(7) Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{bmatrix}$ and $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 4 & 3 & 5 \end{bmatrix}$ find $\alpha^{-1}, \beta\alpha, \alpha\beta$

(8) if $\sigma \in S_6$ ⇒ $\sigma = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{bmatrix}$ ⇒ find σ^{2014}

unit-3

Normal Subgroups & factor G
Rings & integral domain

Important Questions for pass Aspirants

1) State and prove first isomorphism theorem on groups
(or) fundamental theorem of homomorphism

2) Normal Subgroup Test / Necessary & sufficient condition of ~~homomorphism~~ normal subgroup (or)
A SG H of G is normal $\Leftrightarrow$ $xHx^{-1} = H \ \forall x \in H$

3) PT every field is an integral domain

4) ST every finite integral domain is a field.

5) State and prove factor group Theorem

6) $Z(G)$ is a center of a group G
If $\frac{G}{Z(G)}$ is cyclic $\Rightarrow$ PT G is abelian

Remaining Qs

(1) prove that Characterstic of an integral domain is "0" or prime

(2) Find zero divisors in ring Z_{12}

(3) PT every ideal in a ring R is Subring define

(4) PT cancellation law holds good on R $\Leftrightarrow$ R has no divisors

(5) PT SG of an abelian group is normal

(6) PT Center of a group [Z(G)] is always normal

(7) PT intersection of 2 normal SG is normal SG again

(8) PT " " 2 Subrings is again a Subring

(9) D is an Integral domain $\Rightarrow$ PT D[x] is an integral domain

(10) State & prove Subring Test-1 theorem.

A commutative ring R

4th unit

for pass Aspirants

1. State and prove fundamental theorem of Ring Homomorphism or 1st isomorphism theorem for rings

2. $\frac{R}{A}$ is an integral domain $\Leftrightarrow$ A is prime ideal

3. $\frac{R}{A}$ is Field if and only if A is maximal

4. Show that A field has no proper Ideals.

Remaining imp questions of chapter

1. Show that a field has no proper Ideals.

2. If ϕ is a ring homomorphism from a ring R to ring S, then ST $\ker\phi = \{r \in R \mid \phi(r) = 0\}$ is an ideal of R.

(3) Let R be a Commutative Ring with unity and A be an ideal of R then $\frac{R}{A}$ is integral domain $\Leftrightarrow$ prime Ideal

(4) Let ψ be a ring homomorphism from a ring R to ring S. $\Rightarrow$ Kornal $\psi = \{ r [R] \psi(r) = 0 \}$

(3) ST $R[x] / \langle x^2+1 \rangle$ is a field

(5) If F is a Field then prove that every Ideal in $F(x)$ is principal Ideal

(6) ϕ be a ring homomorphism from a Ring R to a ring S $\Rightarrow$ PT ϕ is isomorphism $\Leftrightarrow$ ϕ is onto & $\ker\phi = \{0\}$

(7) If A is an Ideal of a ring R $\Rightarrow$ ST Quotient Ring $\frac{R}{A}$ is Homomorphic Image of R.

(8) State & prove Ideal Test theorem.

Definitions Important